A Shortest Path Search Algorithm Using an Excitable Digital Reaction-Diffusion System

Koichi ITO†, Masahiko HIRATSUKA††, Takafumi AOKI†, Members, and Tatsuo HIGUCHI†††, Fellow

SUMMARY This paper presents a shortest path search algorithm using a model of excitable reaction-diffusion dynamics. In our previous work, we have proposed a framework of Digital Reaction-Diffusion System (DRDS)—a model of a discrete-time discrete-space reaction-diffusion system useful for nonlinear signal processing tasks. In this paper, we design a special DRDS, called an “excitable DRDS,” which emulates excitable reaction-diffusion dynamics and produces traveling waves. We also demonstrate an application of the excitable DRDS to the shortest path search problem defined on two-dimensional (2-D) space with arbitrary boundary conditions.

take words: reaction-diffusion system, nonlinear dynamics, shortest path search, excitable dynamics

1. Introduction

Living organisms can create a remarkable variety of structures to realize intelligent functions. In embryology, the development of patterns and forms is sometimes called Morphogenesis. In 1952, Alan Turing suggested that a system of chemical substances, called morphogens, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis [1]. Recently, model-based studies of morphogenesis employing computer simulations have begun to attract much attention in mathematical biology [2], [3].

From an engineering viewpoint, the insights into morphogenesis provide important concepts for devising a new class of intelligent signal processing functions inspired by biological pattern formation phenomena [4], [5]. From this viewpoint, we have proposed a framework of Digital Reaction-Diffusion System (DRDS)—a discrete-time discrete-space reaction-diffusion dynamical system—for designing signal processing models exhibiting active pattern/texture formation capability. In our previous papers [6], [7], some applications of DRDS to biological texture generation and fingerprint image enhancement/restoration have already been discussed.

The DRDS can simulate a variety of reaction-diffusion dynamics by changing its nonlinear reaction kinetics. This paper describes the design of an excitable DRDS based on FitzHugh-Nagumo-type dynamics [2]; the designed DRDS creates excitable traveling waves exhibiting the following characteristics: (i) the waves propagate with a constant velocity, and (ii) they vanish in collisions with other waves without any other interaction. The goal of this paper is to propose an algorithm for shortest path search in two-dimensional (2-D) space using the excitable DRDS. We define a 2-D map (specifying collision-free space and blocked space) as a boundary condition for the excitable DRDS, and initiate a traveling wave at the starting point in the map. The traveling wave propagates through the map splitting into different groups of wavefronts at branch points. A snapshot of wavefronts represents an equidistant surface measured from the starting point. By tracing back the equidistant surfaces of different time steps, we can find the shortest path from the starting point to any specified destinations.

So far, there are some papers discussing the mechanism of finding the collision-free shortest path in a 2-D map using excitable reaction-diffusion dynamics. In the papers [8]–[13], the real chemical reaction, called Belousov-Zhabotinsky (BZ) reaction, is employed as an excitable medium to generate traveling waves for path finding. The use of real chemical media for performing practical computing tasks has the weakness of limited stability in its operation. Also, the size and complexity of maps that can be handled in chemical computers may be limited. The other related papers basically employ continuous-time models of excitable dynamics, including a PDE (Partial Differential Equation) model [14] and circuit models [11], [15]. All these works focus on the mechanism of generating equidistant surfaces for the given map by using excitable chemical waves and describe only simple examples of small maps. On the other hand, this paper describes a concrete algorithm for shortest path search (including the process of tracing back the equidistant surfaces). The proposed algorithm is based on the discrete-time discrete-space model of DRDS, which is easily implemented in digital computers, and can be applied to arbitrary maps of practical size and complexity.

This paper is organized as follows: Sect. 2 defines an excitable DRDS, and presents some examples of wave propagation in the excitable DRDS. Section 3 describes a shortest path search algorithm using the excitable DRDS. Section 4 demonstrates some experiments for the shortest path

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finding. Section 5 ends with some concluding remarks.

2. Excitable Digital Reaction-Diffusion System

A Digital Reaction-Diffusion System (DRDS)—a model of a discrete-time discrete-space reaction-diffusion dynamical system—can be naturally derived from the original reaction-diffusion system defined in continuous space and time (see [6] for basic mathematical discussions). The general Morphogen DRDS can be obtained as

\[ x(n_0 + 1, n_1, n_2) = x(n_0, n_1, n_2) + R(x(n_0, n_1, n_2)) + D(l * x)(n_0, n_1, n_2), \]  

where \( n_0 \) is a time index, \( n_1 \) and \( n_2 \) are spatial indices, and

\[ x = [x_1, x_2, \cdots, x_M]^T, \]
\[ x_i : \text{concentration of the } i\text{-th morphogen}, \]
\[ R = T_0 \hat{R} = [R_1(x), R_2(x), \cdots, R_M(x)]^T, \]
\[ R_i(x) : \text{nonlinear reaction kinetics for the } i\text{-th morphogen}, \]
\[ T_0 : \text{time sampling interval}, \]
\[ D = \text{diag}(D_1, D_2, \cdots, D_M), \]
\[ \text{diag} : \text{diagonal matrix}, \]
\[ l(n_1, n_2) : \text{diffusion coefficient of the } i\text{-th morphogen}, \]
\[ l(n_1, n_2) = \begin{cases} \frac{1}{T_1^2} & (n_1, n_2) = (-1, 0), (1, 0) \\ \frac{1}{T_2^2} & (n_1, n_2) = (0, -1), (0, 1) \\ -2\left(\frac{1}{T_1^2} + \frac{1}{T_2^2}\right) & (n_1, n_2) = (0, 0) \\ 0 & \text{otherwise}, \end{cases} \]
\[ T_1, T_2 : \text{space sampling intervals}. \]

The operator \( * \) in Eq. (1) denotes the spatial convolution defined as

\[
(l * x)(n_0, n_1, n_2) = \begin{bmatrix}
(l * x_1)(n_0, n_1, n_2) \\
(l * x_2)(n_0, n_1, n_2) \\
\vdots \\
(l * x_M)(n_0, n_1, n_2)
\end{bmatrix} = \sum_{p_1=1}^{1} \sum_{p_2=1}^{1} l(p_1, p_2) x_1(n_0, n_1 - p_1, n_2 - p_2)
\]
\[
= \sum_{p_1=1}^{1} \sum_{p_2=1}^{1} l(p_1, p_2) x_2(n_0, n_1 - p_1, n_2 - p_2)
\]
\[
= \cdots
\]
\[
= \sum_{p_1=1}^{1} \sum_{p_2=1}^{1} l(p_1, p_2) x_M(n_0, n_1 - p_1, n_2 - p_2)
\]

The DRDS described by Eq. (1) can be understood as a three-dimensional nonlinear digital filter. Figure 1 illustrates the computational flow of DRDS with two morphogens \( (M = 2) \). We first store an initial (input) image in \( x_i(0, n_1, n_2) \) at time 0. After computing the dynamics for \( n_0 \) steps, we can obtain the output image from \( x_i(n_0, n_1, n_2) \) at time \( n_0 \).

DRDS can simulate various reaction-diffusion dynamics by changing the nonlinear reaction kinetics \( R \) and its parameters. In this paper, we use the FitzHugh-Nagumo (FHN) model, which is one of the most widely studied excitable models [2]. The two-morphogen FHN-based DRDS, called the excitable DRDS, is defined as follows:

\[
\begin{align*}
   x_1(n_0 + 1, n_1, n_2) &= x_1(n_0, n_1, n_2) + R_1(x_1(n_0, n_1, n_2), x_2(n_1, n_2)) + D_1(l * x_1)(n_0, n_1, n_2) \\
   x_2(n_0 + 1, n_1, n_2) &= x_2(n_0, n_1, n_2) + R_2(x_1(n_0, n_1, n_2), x_2(n_0, n_1, n_2)) + D_2(l * x_2)(n_0, n_1, n_2)
\end{align*}
\]

where

\[
R_1(x_1, x_2) = T_0 \left\{ \frac{1}{k_1} \{x_1[x_1-k_2]|1-x_1|-x_2 \} \right\},
\]
\[
R_2(x_1, x_2) = T_0 \{x_1 - k_3 x_2 \}.
\]

In this paper, we employ the parameter set: \( k_1 = 10^{-3}, k_2 = 10^{-6}, k_3 = 0.1, D_1 = 2, D_2 = 0, T_0 = 10^{-3}, \) and \( T_1 = T_2 = 1 \).

The excitable DRDS exhibits the characteristic behavior of excitable dynamics and generates traveling waves depending on the initial condition. Assume that the initial condition is given by \( x_1(0, n_1, n_2) = x_2(0, n_1, n_2) = 0 \) except for the starting point \( (n_1^2, n_2^2) \). When we give a stimulus above the threshold (\(~0.9\) for the above parameter set) at the starting point, for example, \( x_1(0, n_1^2, n_2^2) = 0.9 \), a traveling wave is initiated from the starting point and propagates with a constant velocity as the time step \( n_0 \) increases.

Consider an excitable DRDS of size \( 128 \times 128 \), where \( n_1 \) and \( n_2 \) are defined as \( 0 \leq n_1 \leq 127 \) and \( 0 \leq n_2 \leq 127 \). Figure 2 shows the wave propagation observed in the snapshots of the first morphogen \( x_1(n_0, n_1, n_2) \). In this example, we first give initial stimuli as \( x_1(0, 32, 64) = x_1(0, 96, 64) = \)
Fig. 2 Wave propagation in the 2-D excitable DRDS: (a) initial condition, (b)–(h) snapshots of wave propagation.

Fig. 3 Time step \( n_0 \) when the traveling wave starting at the point \((0, 0)\) arrives at the point \((n_1, 0)\).

In the above example, we can observe two important characteristics of excitable waves: (i) the waves propagate with a constant velocity and (ii) they vanish in collisions with other waves. As for the item (i) above, Fig. 3 shows the arrival time (in time step \( n_0 \)) of a traveling wave as a function of the distance (in pixels) from the starting point. Note that the arrival time of a traveling wave is defined as the time when the concentration \( x_1 \) exceeds the threshold value 0.9 for the first time. We can observe that the traveling wave propagates with a constant velocity of 0.11 pixels per a unit time step. These features suggest a unique algorithm for the shortest path search problem as described in Ref. [8], where snapshots of real propagating chemical waves are considered as a collection of equidistant surfaces and are useful for finding the shortest path from the starting point to any specified points in 2-D space.

3. Shortest Path Search Algorithm

The original experiment of the shortest path search using actual chemical waves is found in Ref. [8], where the optimal pathways were determined by the collection of time-lapse position information on actual chemical waves propagating through 2-D mazes prepared with the Belousov-Zhabotinsky (BZ) reaction. Inspired by the natural computing using chemical wave propagation, we propose a shortest path search algorithm using the excitable DRDS designed in the above section. The proposed algorithm employs the excitable DRDS for wavefront generation and performs the traceback of traveling wavefronts to find the shortest paths.

Figures 4 (a)–(c) show the wave propagation in a 2-D excitable DRDS of the size 128 \( \times \) 128 pixels. Note that we employ the boundary condition defined by obstacles in the map, where we set fixed concentrations \( x_1 = 0 \) and \( x_2 = 0 \) for obstacle locations. In Fig. 4(d), 29 snapshots of wavefronts of the first morphogen \( x_1 \) are superimposed at every 100-step intervals to form a composite image. Each wavefront represents a set of equidistant locations measured from the starting point, and hence we can derive the shortest path by tracing back the history of wavefront position from the goal to the starting point.

The proposed algorithm consists of two operations: Forward Operation and Backward Operation. Forward Operation is to generate a traveling wave in the excitable DRDS and record snapshots of equidistant wavefronts with specific time intervals. Backward Operation, on the other hand, is to trace back the wavefronts from the goal to the starting point to find the shortest pathways. We can obtain the shortest path by connecting every pair of two points on the adjacent equidistant wavefronts with the shortest distance. Figures 5 and 6 show the detailed algorithms for Forward and Back-
3.1 Forward Operation

We illustrate here the detailed algorithm for Forward Operation. The inputs of Forward Operation are a starting point \((n_1^0, n_2^0)\), a goal \((n_1^G, n_2^G)\), and a map (with obstacle information).

\[ W(n_0) = \text{a list of pixel positions at the boundary condition defined by the map} \]

\[ x_1(0, n_1, n_2) = x_2(0, n_1, n_2) = 0 \]

\[ W(n_0) \text{ stores the list of position at which the traveling wave exists} \]

\[ n_0^G \text{ is the time step when the traveling wave arrives at the goal} \]

\[ n_1(0, n_1 + 1) > n_1^G \text{ at the initial value of the first morphogen} \]

\[ W(0) \text{ stores only the starting position} \]

\[ n_1^G, n_2^G \text{ at first} \]

\[ \text{In this step, we compute the excitable DRDS (Eq. (2))} \]

\[ W(0) \text{ stores a list of pixel positions (}) \]

\[ \text{at the initial value of the first morphogen} \]

\[ \text{higher than the threshold value 0.9. Hence, W(0) stores only the starting} \]

\[ \text{position} \]

\[ \text{at first.} \]

\[ \text{for one step assuming the boundary condition defined by the map} \]

\[ \text{and derive the updated values} \]

\[ \text{until the traveling wave arrives at the goal} \]

\[ \text{the initial condition is given by} \]

\[ \text{the starting point} \]

\[ W(0) \text{ stores only the starting} \]

\[ \text{the traveling wave arrives at the goal} \]

\[ \text{the snapshots of wave propagation in x1(n0, n1, n2), (d) superposition of traveling waves in x1(n0, n1, n2)} \]

\[ \text{the starting point} \]

\[ \text{the map and derive the updated values} \]

\[ \text{the map defines the obstacle region. The outputs of Forward Operation are} \]

\[ \text{every 100 steps.} \]
ward Operation uses the set of wavefront records \(\Delta\), a resolution of the time step interval for Backward Operation (\(\Delta = 4\) in our experiments); 

**Output**

Paths: a list of points (with their time index) on the shortest path from \((n_0^G, n_1^G, n_2^G)\) to \((0, n_1^S, n_2^S)\);

1. begin
2. Search \(\leftarrow (n_0^G, n_1^G, n_2^G)\);
3. Paths \(\leftarrow \) Backtrace\(\leftarrow\)Search
4. end.

**function** Backtrace\(\leftarrow\)Search

1. begin
2. \((n_0^G, n_1^G, n_2^G) \leftarrow\) the last element of Search;
3. if \(n_0^G - \Delta \leq 0\) then
4. Search \(\leftarrow\) append\(\leftarrow\)Search, \(\leftarrow (0, n_1^G, n_2^G)\);
5. return Search
6. else
7. begin
8. \(C \leftarrow \) get the points in \(W(n_0^G - \Delta)\) that have the shortest distance from \((n_1^G, n_2^G)\);
9. \(C \leftarrow \) BranchDetection\(\leftarrow\)C;
10. Search \(\leftarrow\) \(\bigcup\) Backtrace\(\leftarrow\)append\(\leftarrow\)Search, \(\leftarrow (0, n_0^G - \Delta, n_1^G, n_2^G)\);
11. return Search
12. end
13. end.

**function** BranchDetection\(\leftarrow\)C

1. begin
2. \(C' \leftarrow \) get the points in \(W(n_0^G - 2\Delta)\) that have the shortest distance from each point in \(C\) (see Fig. 7);
3. Calculate the distance between the two (or more) points in \(C'\);
4. if the calculated distance is less than \(\Delta\) then
5. Replace the points in \(C\) with their middle position;
6. return \(C\)
7. end.

![Fig. 6](image)

Algorithm for Backward Operation.

is recorded in \(n_0^G\).

### 3.2 Backward Operation

We describe here the detailed algorithm for Backward Operation. The inputs for Backward Operation are the starting point \((n_1^G, n_2^G)\), the goal \((n_1^G, n_2^G)\), wavefront records \(W(n_0)\), the arrival time at the goal \(n_0^G\) and \(\Delta\) that defines the resolution of time step interval for Backward Operation. Backward Operation uses the set of wavefront records \(W(n_0^G)\), \(W(n_0^G - \Delta)\), \(W(n_0^G - 2\Delta)\), \(W(n_0^G - 3\Delta)\), \(W(n_0^G - 4\Delta)\), \ldots to trace the shortest path recursively from the goal to the starting point. We use \(\Delta = 4\) in our experiments. The outputs of Backward Operation consists of three sections: (i) initial setting, (ii) Backtrace, and (iii) BranchDetection.

**i) Initial Setting**

The search list Search is initialized to \((n_0^G, n_1^G, n_2^G)\) as a possible shortest path candidate. Backtrace procedure described below is then started with the search list Search.

**ii) Backtrace Procedure**

In this step, we find the point on the shortest path by tracing back the wavefront from the goal to the starting point. First, we get a point \((n_0^G, n_1^G, n_2^G)\) from the last element of the search list Search. In the case when \(n_0^G - \Delta \leq 0\), we add \((0, n_1^G, n_2^G)\) (i.e., the starting point) to the search list Search and return Search. In other cases (i.e., \(n_0^G - \Delta > 0\)), we load \(W(n_0^G - \Delta)\), the set of wavefront points at time \(n_0^G - \Delta\), and find the set of points \(C(\subseteq W(n_0^G - \Delta))\) having the shortest distance from \((n_1^G, n_2^G)\) in 2-D space. To select the points to be stored in \(C\), we need to perform (iii) BranchDetection procedure described in the next paragraph. There is a possibility that the shortest path has multiple branches at time \(n_0^G - \Delta\) and, in such a case, we store multiple points into \(C\). We add each point \((n_0^G, n_1^G, n_2^G)\) in \(C\) with its time index \(n_0^G - \Delta\) to the search list Search, and perform Backtrace procedure. The Backtrace procedure is performed recursively until the time step \(n_0^G - \Delta\) becomes less than or equal to 0 (in other words, the search point arrives at the starting point).

**iii) BranchDetection Procedure**

There is a possibility that \(C\) (the set of wavefront points that are closest to \((n_0^G, n_1^G, n_2^G)\)) contains multiple points, i.e.,
\[ |C| \geq 2. \] In such a case, we need to check whether the points in \( C \) are true branches or not. BranchDetection procedure further traces the wavefront backward to verify whether the set of points in \( C \) represent appropriate branches on the shortest pathways or not. For every point in \( C \), we first find the points in \( W(n_0^k - 2\Delta) \) that have the shortest distance from the point and store the detected points in \( C' \). This process is schematically illustrated in Fig. 7. We calculate the distance between the two points in \( C' \) and classify the points in \( C' \) into groups based on the calculated distances. If the distance between the two points in \( C' \) is less than a threshold value (4 pixels in our experiments), these points are considered to be in the same group (i.e., in the same branch). We can then classify the points in \( C \) into groups according to the grouping for \( C' \). To reduce computational complexity, we replace the points in \( C \) belonging to the same group with their middle position. The size of \( C \) directly corresponds to the number of branches at time \( n_0^k - \Delta \). If there exists multiple groups in \( C \), we can consider that the shortest path is divided into multiple pathways.

Backward Operation mentioned above can find all the possible shortest paths from the starting point to the given goal.

We can briefly confirm that the proposed algorithm can find the shortest path from the starting point \((n_k^1, n_k^2)\) to the goal \((n_0^k, n_0^k)\) as follows:

- Forward Operation computes the equidistant surfaces from the starting point for all the time steps \( n_0 \) (\( 0 \leq n_0 \leq n_0^k \)). When Forward Operation terminates, the equidistant surface (at time \( n_0^k \)) arrives at the goal.
- When we start Backward Operation, clearly the shortest path from the goal to the equidistant surface at time \( n_0^k \) is obtained (since the goal is on the equidistant surface).
- After the first iteration of Backward Operation (i.e., after the first call of Backtrace function), the shortest path from the goal to the equidistant surface at time \( n_0^k - \Delta \) is obtained, since the Backtrace function is designed to find the point (on the equidistant surface at time \( n_0^k - \Delta \)) having minimum distance from the goal.
- Similarly, after the second call of Backtrace function, the shortest path from the goal to the equidistant surface at time \( n_0^k - 2\Delta \) is obtained.
- This process is repeated until the Backward Operation finds the shortest path from the goal to the equidistant surface at time 0. Note that the equidistant surface at time 0 is equivalent to the starting point, and hence the shortest path from the goal to the starting point is obtained when finishing Backward Operation.

### 4. Experiments

This section presents some experiments of shortest path search using the proposed algorithm under three different conditions: (i) 2-D free space, (ii) map with symmetric obstacles, and (iii) maps with complicated obstacles.

In our proposed algorithm, the system parameters \( k_1, k_2, k_3, D_1, D_2, T_0, T_1, T_2 \) and \( \Delta \) are adjusted in advance through a set of experiments. Basically, we do not need to change these parameters depending on the given problem. Our strategy of determining the adequate values of system parameters is stated as follows: (i) determine the parameters \( k_1, k_2, k_3 \) for reaction kinetics of DRDS by analyzing its phase-plane diagram so as to exhibit excitatory behavior, (ii) determine the parameters \( D_1, D_2, T_0, T_1 \) and \( T_2 \) so as to generate the equidistant surfaces with a sufficient level of accuracy, and (iii) determine the time step interval \( \Delta \) for Backward Operation, which controls the accuracy of the shortest path estimation for the given 2-D map. In our experiments, we employ the system parameters as \( k_1 = 10^{-3}, k_2 = 10^{-6}, k_3 = 0.1, D_1 = 2, D_2 = 0, T_0 = 10^{-3}, T_1 = T_2 = 1 \) and \( \Delta = 4 \).

(i) 2-D Free Space

Figure 8 shows the experimental result on the 2-D free space. In this example, the map is \( 512 \times 512 \) free space, the starting point \((n_0^1, n_0^2)\) is \((3, 3)\), and the goal \((n_0^k, n_0^k)\) is \((509, 509)\). The traveling wave initiated from the starting point propagates from upper left to lower right drawing a circular pattern in Forward Operation. The shortest path from the starting point to the goal is obtained as a straight line in Backward Operation. The obtained path is obviously the shortest as shown in Fig. 8.

(ii) Map with Symmetric Obstacles

Figure 9 shows the experimental result on the map with symmetric obstacles. In this experiment, we obtain two shortest paths, since obstacles are symmetric to the line between the starting point and the goal. As is observed in this experiment, we can obtain two shortest paths which have equal distances.

(iii) Maps with Complicated Obstacles

Figure 10 shows typical examples of shortest path search in complicated mazes, where multiple goals are specified in advance. All the obtained paths avoid obstacles and are the shortest paths.

In the above experiments, we can observe that all the obtained paths from the starting point to the goal are shortest in terms of Euclidean distance.

We discuss here the computational complexity of the
proposed algorithm. The computation time of Forward Operation is proportional to $N_1 \times N_2 \times n_G^0$, and that of Backward Operation is proportional to $n_G^0$. This means that the total computation time depends directly on the size of 2-D map and the distance between the starting point and the goal. Table 1 summarizes measured computation time of the proposed algorithm for various size of 2-D maps, where the experimental results clearly show the predicted tendency. In this experiment, MATLAB ver. 6.5.1 on Pentium 4 (3.2 GHz) is employed for programming, and thus the computation time can be drastically reduced by translating the MATLAB code to other implementation-oriented languages such as C/C++.

![Fig. 8](image1.png) **Fig. 8** Shortest path search on the 2-D free space (the wavefronts in $x_1(n_0, n_1, n_2)$ are superimposed on the map every 80 steps).

![Fig. 9](image2.png) **Fig. 9** Shortest path search on the map with symmetric obstacles (the wavefronts in $x_1(n_0, n_1, n_2)$ are superimposed on the map every 70 steps).

![Fig. 10](image3.png) **Fig. 10** Shortest path search on the maps with complicated obstacles (the wavefronts in $x_1(n_0, n_1, n_2)$ are superimposed on the map every 74 steps). $S$ is a starting point $(n_1^g, n_2^g)$ and $G_i (i = 1, \cdots, 9)$ are goals $(n_1^i, n_2^i)$.

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<th>$(n_1^i, n_2^i)$</th>
<th>$n_G^0$</th>
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<th>Backward Operation</th>
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5. Conclusion and Discussions

This paper presents a shortest path search algorithm in 2-D space using the excitable Digital Reaction-Diffusion System (DRDS). The traveling wave generated by the excitable DRDS has two significant features: (i) propagation with a constant velocity and (ii) annihilation in collisions with other waves. These features are effectively used to find the shortest paths in 2-D maps with various obstacles.

An alternative approach for solving large-scale shortest path problems may be to employ real chemical reactions (such as Belousov-Zhabotinsky (BZ) reaction [9], [11]) for generating traveling waves or to employ electric circuit models of excitable dynamics [16], [17]. Such approaches may have advantages of massively parallel operation for large-scale problems, and are expected to provide interesting alternatives to conventional digital computers. However, major drawbacks of using real chemical media or electric circuits are the lack of flexibility in tuning system parameters including various constants for reaction-diffusion dynamics, and the limited programmability for 2-D maps on which the shortest-path problem is solved.

On the other hand, an important advantage of the DRDS-based approach is its flexibility and programmability. We can easily modify system parameters as well as boundary conditions depending on the given problem. The proposed algorithm can be applied to various navigation tasks defined in 2-D space, and could be extended also to higher-dimensional space. A major weakness of the DRDS-based approach is its computational complexity. Clearly, this can be addressed with parallel processing technology by fully exploiting massive parallelism in DRDS computation.

The shortest path search problem discussed in this paper can be considered as an instance of discrete optimization problems—optimization problems defined over discrete structures. A subset of areas covered by discrete optimization includes transportation (vehicle routing, job scheduling, packing), industrial planning (production planning, machine scheduling, inventory management), finance (investment planning, portfolio management), molecular biology (DNA alignment), system design (VLSI design, processor scheduling, network design), etc. It is possible to define the shortest path search problem discussed in this paper in terms of discrete optimization. There are many approaches for solving discrete optimization problems including optimal methods (branch and bound, dynamic programming, etc.) and heuristics (local search, simulated annealing, genetic algorithms, etc.). Note that the DRDS-based approach is not a general-purpose framework for discrete optimization. A natural question may arise here: What class of problems could be handled by DRDS-based approach? The excitable DRDS can generate accurate equidistant surfaces from a given starting point, and hence it may be applied to some problems in computational geometry, in which the equidistant surfaces play an important role to solve the problems. Some examples of applications are Voronoi diagram formation and thinning or skeletonization of binary images. Detailed characterization of possible applications is being left for future study.

References

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