This paper presents a digital reaction-diffusion system (DRDS) – a model of a discrete-time discrete-space reaction-diffusion dynamical system – for designing new image processing algorithms inspired by biological pattern formation phenomena. The original idea is based on the Turing’s model of pattern formation, which is widely known in mathematical biology. The DRDS is useful for generating biological textures, patterns and structures. This paper describes the design of DRDS for image processing tasks, such as biological texture generation, fingerprint image restoration and shortest path search.

1. INTRODUCTION

Living organisms can create a remarkable variety of patterns and forms from genetic information. In embryology, the development of patterns and forms is sometimes called **Morphogenesis**. In 1952, Alan Turing suggested that a system of chemical substances, called **morphogens**, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis [1]. From an engineering viewpoint, the insights into morphogenesis provide important concepts for devising a new class of intelligent signal processing algorithms inspired by biological pattern formation phenomena [2]–[4].

In general, most of computational models of biological pattern formation are described by continuous-time continuous-space reaction-diffusion equations, and hence can not be directly handled by the theory of digital signal processing. Addressing this problem, we propose a Digital Reaction-Diffusion System (DRDS) — a model of a discrete-time discrete-space reaction-diffusion dynamical system, which is useful for designing new types of signal processing algorithms based on biological pattern formation mechanism [5], [6]. Using the DRDS, mathematical models of morphogenesis can be understood by multidimensional digital signal processing theory. In this paper, we present the basic framework of DRDS, and its application to computer graphics, fingerprint image restoration and shortest path search.

2. DIGITAL REACTION-DIFFUSION SYSTEM (DRDS)

A Digital Reaction-Diffusion System (DRDS) – a model of a discrete-time discrete-space reaction-diffusion dynamical system – can be naturally derived from the original reaction-diffusion system defined in continuous space and time. The general $M$-morphogen reaction-diffusion system with two-dimensional (2-D) space indices $(r_1, r_2)$ is written as

$$
\frac{\partial \tilde{x}(t, r_1, r_2)}{\partial t} = \tilde{R}(\tilde{x}(t, r_1, r_2)) + \tilde{D} \nabla^2 \tilde{x}(t, r_1, r_2),
$$

where

$\tilde{x} = [\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_M]^T$,

$\tilde{x}_i$: concentration of the $i$-th morphogen,

$\tilde{R}(\tilde{x}) = [\tilde{R}_1(\tilde{x}), \tilde{R}_2(\tilde{x}), \ldots, \tilde{R}_M(\tilde{x})]^T$,

$\tilde{R}_i(\tilde{x})$: reaction kinetics for the $i$-th morphogen,

$\tilde{D} = \text{diag} [\tilde{D}_1, \tilde{D}_2, \ldots, \tilde{D}_M]$, 

$\text{diag}$: diagonal matrix,

$\tilde{D}_i$: diffusion coefficient of the $i$-th morphogen.

We now sample a continuous variable $\tilde{x}$ in (1) at the time sampling interval $T_0$, and at the space sampling intervals $T_1$ and $T_2$. Assuming discrete time-index to be given by $n_0$ and discrete space
Using this discretization, the general DRDS can be understood as a 3-D nonlinear digital filter. We first store an initial (input) image in a specific morphogen, say \( x_i(0, n_1, n_2) \), at time 0. After computing the dynamics for \( n_0 \) steps, we can obtain the output image from one of the \( M \) morphogens, say \( x_i(n_0, n_1, n_2) \), at time \( n_0 \). In general, linear digital filters with guaranteed stability are widely used in many signal processing applications. In our application, however, we employ the DRDS with nonlinear reaction kinetics \( R(x) \) satisfying the diffusion-driven instability condition \([5]\). In this case, DRDS becomes an unstable 3-D nonlinear digital filter having significant pattern formation capability.

In this paper, we use the two-morphogen DRDS \((M=2)\) with the Brusselator reaction kinetics, which is one of the most widely studied chemical oscillators \([7]\). The two-morphogen Brusselator-based DRDS is defined as follows:

\[
\begin{align*}
    \begin{bmatrix}
        x_1(n_0+1, n_1, n_2) \\
        x_2(n_0+1, n_1, n_2)
    \end{bmatrix}
    &=
    \begin{bmatrix}
        R_1(x_1(n_0, n_1, n_2), x_2(n_0, n_1, n_2)) \\
        R_2(x_1(n_0, n_1, n_2), x_2(n_0, n_1, n_2))
    \end{bmatrix}
    +
    \begin{bmatrix}
        D_1(l \ast x_1(n_0, n_1, n_2)) \\
        D_2(l \ast x_2(n_0, n_1, n_2))
    \end{bmatrix},
\end{align*}
\]

where

\[
\begin{align*}
    R_1(x_1, x_2) &= T_0 \left\{ k_1 - (k_2 + 1) x_1 + x_1^2 x_2 \right\}, \\
    R_2(x_1, x_2) &= T_0 \left( k_2 x_1 - x_1^2 x_2 \right).
\end{align*}
\]

In this paper, we consider the parameter set: \( k_1 = 2, \ k_2 = 4, \ T_0 = 0.01, \ D_1 = T_0 \) and \( D_2 = 5T_0 \). In this case, the DRDS shows the spot pattern formation from random initial concentration as shown in Fig. 1. By changing the parameters, we can generate various biological patterns.

### 3. FINGERPRINT RESTORATION

In this section, we show the fingerprint image enhancement/restoration using DRDS.
becomes the initial input gerprint image into $[1,3]$ range. The scaled image set. Hence, we first scale the $[0,255]$ gray-scale fin-
dynamics (4) has the equilibrium $(t)$.

Note that spatial sampling parameters $T_1$ and $T_2$

should be adjusted according to the inherent spa-
tion. After $n$ boundary condition is employed for computing the
boundary value problem and 1

$b_i \leq 2$ in the case of given parameter

$x$ means to scale the initial fingerprint image in

to produce the final output.

Figure 2: Enhancement of a fingerprint image.

The two-morphogen DRDS (4) can be used to
enhance fingerprint patterns [5]. We first set the
initial fingerprint image in $x_1(0, n_1, n_2)$, at time 0.
Note that spatial sampling parameters $T_1$ and $T_2$

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In order to solve this problem, the next section
takes account of the local orientation of ridge flow.

Some broken ridge lines in processing fingerprint
images as shown in Fig. 2(e), since it does not

fully isotropic diffusion term of (4) often produces

tion, however, shows that the DRDS with a spa-
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Figure 3: Generation of the orientation mask.

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Figure 3: Generation of the orientation mask.
Algorithm for the adaptive DRDS with hierarchical orientation estimation

1. begin
2.   \[ p := 2; \ \{ \text{initialize the image partitioning factor} \} \]
3. while time step \( n_0 \) equals to 500 do
4.   begin
5.     if \( p \) is less than 9 then
6.     begin
7.       partition the input image into \( p^2 \) sub-images;
8.       generate independent orientation masks for \( p^2 \) sub-images;
9.       run the adaptive DRDS (Eq. (5)) for 10 time steps;
10. \( p := p + 1 \)
11. end
12. else
13. begin
14.     generate independent orientation masks for all pixels;
15.     run the adaptive DRDS (Eq. (5)) for 10 time steps
16. end
17. end
18. end.

Figure 4: Algorithm for the adaptive DRDS with hierarchical orientation estimation.

Fourier transform of the local image around the pixel \((n_1, n_2)\), (ii) extract the dominant ridge orientation \(\theta\) from the transformed image, (iii) generate a mask pattern \(H_{1,m_1m_2}^{n_1}(j\omega_1, j\omega_2)\) having the orientation \(\theta\) in frequency domain as

\[
H_{1,m_1m_2}^{n_1}(j\omega_1, j\omega_2) = \begin{cases} 
1 & \text{for unstable frequency band} \\
2 & \text{(black pixels in Fig. 3(ii))} \\
2 & \text{otherwise,}
\end{cases}
\]

and (iv) take the inverse Fourier transform to obtain the orientation mask \(h_{1,m_1m_2}^{n_1}(n_1, n_2)\). The orientation mask \(h_{2,m_1m_2}^{n_1}(n_1, n_2)\) for the second morphogen, on the other hand, has the value 1 at the center \((n_1, n_2) = (0, 0)\) and equals to 0 for the other coordinates \((n_1, n_2)\). Thus, the dynamics for the morphogen \(x_2(n_0, n_1, n_2)\) does not take account of the local orientation.

In practical situation, it is difficult to obtain the exact orientation masks from blurred fingerprints directly. Addressing this problem, we estimate local orientation masks recursively using a coarse-to-fine approach as shown in Fig. 4. This restoration algorithm starts with rough estimation of local orientation for four sub-images \((p = 2)\).

The image partitioning factor \(p\) gradually increases as restoration step \(n_0\) increases. We can obtain pixel-wise orientation masks \(h_{m_1m_2}^{n_1}(n_1, n_2)\) after 80 time steps. This simple strategy makes possible significant improvement in the precision of orientation estimation.

The problem considered here is to restore the original fingerprint image from its “subsampled” image. For this purpose, we generate a subsampled fingerprint image from the original image as follows: (i) partition the original image into \(R \times S\)-pixel rectangular blocks, and (ii) select one pixel randomly from every block and eliminate all the other pixels (set 127, middle gray-level, to the pixels). The image thus obtained has the same size as the original image, but the number of effective pixels is reduced to \(1/(R \times S)\). Figure 5 shows the original image, the subsampled image \((n_0 = 0)\) and restored images at \(n_0 = 100, 200\) and 400, respectively, for the case of \(1/(6 \times 6)\) subsampling. We can observe that the fingerprint pattern is reconstructed from the subsampled image gradually as time step \(n_0\) increases.

4. SHORTEST PATH SEARCH

This section presents the shortest path search using the DRDS with FitzHugh-Nagumo (FHN) reaction kinetics, which is one of the most widely studied excitable model [7]. We shall define the FHN-based reaction kinetics for DRDS as

\[
R_1(x_1, x_2) = \frac{T_0}{k_1}(x_1(x_1-k_2)(1-x_1)-x_2), \\
R_2(x_1, x_2) = T_0(x_1-k_3x_2). \\
\]

In this paper, we employ the parameter set: \(k_1 = 10^{-3}, k_2 = 10^{-6}, k_3 = 0.1, D_1 = 40, D_2 = 0, T_0 = 10^{-3} \) and \(T_1 = T_2 = 1\).

The excitable DRDS exhibits spatio-temporal wave patterns. The excitable DRDS indicates the most important features of FHN dynamics; waves exhibit constant velocities and annihilate in collisions with boundaries or other waves. These features suggest a unique algorithm for the shortest path planning problems as described in [8]. Figure 6 shows a superposition of an excitable wave propagating through two-dimensional complex maze. A single propagating wave generates a map for finding the shortest path from a start point to any specified point in these maze.
5. CONCLUSION

This paper presents a digital reaction-diffusion system (DRDS) – a model of a discrete-time discrete-space reaction-diffusion dynamical system – useful for signal processing and computer graphics applications. We are expecting that the framework of DRDS may provide a theoretical foundation of digital morphogenesis, that is, a technique for applying the principle of biological pattern formation phenomena to many engineering problems.

REFERENCES


