The 2004 International Technical Conference on Circuits/Systems, Computers and Communications (ITC-CSCC2004)
Hotel Taikanso, Sendai/Matsushima, Miyagi-Pref., JAPAN July 6-8, 2004

Design of an Excitable Digital Reaction-Diffusion System for Shortest Path Search

Koichi Ito†*, Takafumi Aoki†, and Tatsuo Higuchi‡

†Graduate School of Information Sciences,
Tohoku University
Aoba-yama 05, Sendai, 980-8579 Japan.
Phone: +81-22-217-7169, Fax: +81-22-263-9308,
E-mail: ito@aoki.ecei.tohoku.ac.jp
*Research Fellow of the Japan Society for the Promotion of Science

‡Department of Electronic Engineering,
Faculty of Engineering,
Tohoku Institute of Technology
Sendai, 982-8577 Japan.

Abstract: Recently, model-based studies of morphogenesis employing computer simulations have begun to attract much attention in mathematical biology. Morphogenesis can be described and modeled mathematically by reaction-diffusion system. We proposed a Digital Reaction-Diffusion System (DRDS) — a model of a discrete-time discrete-space reaction-diffusion system for nonlinear signal processing tasks. Example applications include enhancement and restoration of fingerprint images as well as generation of biological textures for computer graphics applications. In this paper, we present the design of a special DRDS that emulates characteristic behavior of excitable dynamics, and demonstrate its application to a shortest path search problem.

1. Introduction

Living organisms can create a remarkable variety of structures to realize their intelligent functions. In embryology, the development of patterns and forms is sometimes called *Morphogenesis*. In 1952, Alan Turing suggested that a system of chemical substances, called *morphogens*, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis [1]. Recently, model-based studies of morphogenesis employing computer simulations have begun to attract much attention in mathematical biology [2], [3].

From an engineering viewpoint, the insights into morphogenesis provide important concepts for devising a new class of intelligent signal processing functions inspired by biological pattern formation phenomena [4], [5]. From this viewpoint, we have proposed a framework of *Digital Reaction-Diffusion System* (DRDS) – a discrete-time discrete-space reaction-diffusion dynamical system – for designing signal processing models exhibiting active pattern/texture formation capability, and applied DRDS to the biological texture generation and to the fingerprint image enhancement/restoration [6], [7].

The DRDS can simulate various reaction-diffusion dynamics by changing the nonlinear reaction function and its parameters. In this paper, we newly design a FitzHugh-Nagumo-type excitable DRDS, which can create excitable traveling waves having the following features: (i) the waves propagate with constant velocities, and (ii) they vanish in collisions with the other waves. We also propose an algorithm of shortest path search in two-dimensional space using the excitable DRDS. We first define a map on the two-dimensional DRDS, and initiate a traveling wave at the starting point in the map. The traveling waves propagate through the map and generate the equidistant surfaces. By using the equidistant surfaces, we can search the shortest path from the starting point to any specified point in the map. On the basis of this idea, this paper presents a DRDS-based shortest path search algorithm, which is useful for solving navigation tasks in arbitrary two-dimensional maps with obstacles.

2. Excitable Digital Reaction-Diffusion System

A Digital Reaction-Diffusion System (DRDS) – a model of a discrete-time discrete-space reaction-diffusion dynamical system – can be naturally derived from the original reaction-diffusion system defined in continuous space and time (see [6] for detailed mathematical formulation). The general M-morphogen DRDS can be obtained as

$$\begin{aligned}&\boldsymbol{x}(n_0+1, n_1, n_2)\\ &\quad= \boldsymbol{x}(n_0, n_1, n_2)+\boldsymbol{R}(\boldsymbol{x}(n_0, n_1, n_2))\\ &\qquad+\boldsymbol{D}(l * \boldsymbol{x})(n_0, n_1, n_2),\end{aligned} \tag{1}$$

where

$$\begin{aligned}\boldsymbol{x} &= [x_1, x_2, \cdots, x_M]^T,\\ &\quad x_i : \text{concentration of the } i\text{-th morphogen},\\ \boldsymbol{R} &= T_0\tilde{\boldsymbol{R}} = [R_1(\boldsymbol{x}), R_2(\boldsymbol{x}), \cdots, R_M(\boldsymbol{x})]^T,\\ &\quad R_i(\boldsymbol{x}) : \text{reaction kinetics for the } i\text{-th morphogen},\\ \boldsymbol{D} &= diag[D_1, D_2, \cdots, D_M],\\ &\quad diag : \text{diagonal matrix},\\ &\quad D_i : \text{diffusion coefficient of the } i\text{-th morphogen},\end{aligned}$$

$$l(n_1, n_2)= \begin{cases} \frac{1}{T_1^2} & (n_1, n_2) = (-1, 0), (1, 0)\\ \frac{1}{T_2^2} & (n_1, n_2) = (0, -1), (0, 1)\\ -2(\frac{1}{T_1^2} + \frac{1}{T_2^2}) & (n_1, n_2) = (0, 0)\\ 0 & \text{otherwise,}\end{cases}$$

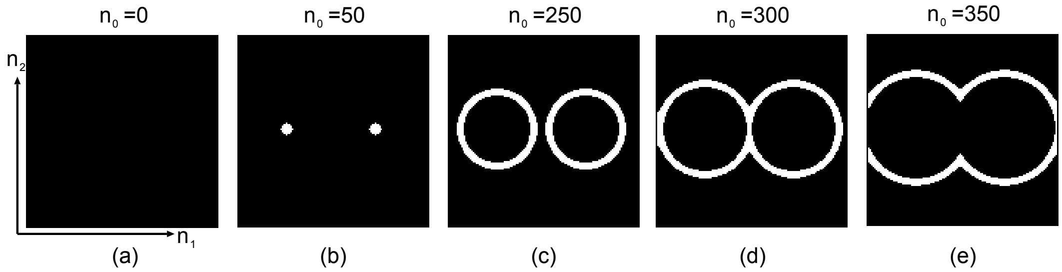

Figure 1. Wave propagation in two-dimensional excitable DRDS: (a) initial condition, (b)–(e) wave propagation.

and $*$ is the spatial convolution operator defined as

$$(l * \boldsymbol{x})(n_0, n_1, n_2) = \begin{bmatrix} (l * x_1)(n_0, n_1, n_2) \\ (l * x_2)(n_0, n_1, n_2) \\ \vdots \\ (l * x_M)(n_0, n_1, n_2) \end{bmatrix} = \begin{bmatrix} \sum_{p_1=-1}^{1} \sum_{p_2=-1}^{1} l(p_1, p_2) x_1(n_0, n_1 - p_1, n_2 - p_2) \\ \sum_{p_1=-1}^{1} \sum_{p_2=-1}^{1} l(p_1, p_2) x_2(n_0, n_1 - p_1, n_2 - p_2) \\ \vdots \\ \sum_{p_1=-1}^{1} \sum_{p_2=-1}^{1} l(p_1, p_2) x_M(n_0, n_1 - p_1, n_2 - p_2) \end{bmatrix}.$$

DRDS can simulate various reaction-diffusion dynamics by changing the nonlinear reaction kinetics and its parameters. In this paper, we use the FitzHugh-Nagumo (FHN) model, which is one of the most widely studied excitable models [2]. The two-morphogen FHN-based DRDS called the *excitable* DRDS is defined as follows:

$$\begin{bmatrix} x_1(n_0{+}1, n_1, n_2) \\ x_2(n_0{+}1, n_1, n_2) \end{bmatrix} = \begin{bmatrix} x_1(n_0, n_1, n_2) \\ x_2(n_0, n_1, n_2) \end{bmatrix} + \begin{bmatrix} R_1(x_1(n_0, n_1, n_2), x_2(n_0, n_1, n_2)) \\ R_2(x_1(n_0, n_1, n_2), x_2(n_0, n_1, n_2)) \end{bmatrix} + \begin{bmatrix} D_1(l * x_1)(n_0, n_1, n_2) \\ D_2(l * x_2)(n_0, n_1, n_2) \end{bmatrix}, \qquad (2)$$

where

$$R_1(x_1, x_2) = T_0 \left[\frac{1}{k_1} \{ x_1(x_1 - k_2)(1 - x_1) - x_2 \} \right],$$
$$R_2(x_1, x_2) = T_0 \left(x_1 - k_3 x_2 \right).$$

In this paper, we employ the parameter set: $k_1 = 10^{-3}$, $k_2 = 10^{-6}$, $k_3 = 0.1$, $D_1 = 40$, $D_2 = 0$, $T_0 = 10^{-3}$, and $T_1 = T_2 = 1$.

The excitable DRDS exhibits the excitable behavior and generates traveling waves depending on the initial condition. Assume that the initial condition is given by $x_1(0, n_1, n_2) = x_2(0, n_1, n_2) = 0$ except for the starting point (n_1^S, n_2^S). When we give a certain amount of stimulus above the threshold (~ 0.9 assuming the above parameter set) at the starting point, for example, $x_1(0, n_1, n_2) = 0.9$, a traveling wave is initiated from the starting point and propagates with a constant velocity as the time step n_0 increases.

Figure 1 shows the wave propagation in the two-dimensional excitable DRDS. In this example, we first give initial stimuli as $x_1(0, 64, 32) = x_1(0, 64, 96) = 0.9$, where $0 \leq n_1, n_2 \leq 127$ (Fig. 1 (a)). The traveling waves spread in a circular pattern and vanish in collisions with the other wave as show in Figs. 1(b)–(e). In this example, we can observe two important characteristics of excitable waves: (i) propagating with constant velocities and (ii) vanishing in collisions with boundaries and other waves. These features suggest a unique algorithm for the shortest path search problem as described in [8], where snapshots of propagating waves are considered as equidistant surface from the starting point and used for finding the shortest path from the starting point to any specified point in two-dimensional space.

3. Shortest Path Search Algorithm

The original idea of the shortest path search using actual chemical waves could be found in the reference paper [8], where the optimal pathways were determined by the collection of time-lapse position information on actual chemical waves propagating through two-dimensional mazes prepared with the Belousov-Zhabotinsky (BZ) reaction. Inspired by the natural computing using chemical wave propagation, we propose a shortest path search algorithm using the excitable DRDS. The proposed algorithm employs the excitable DRDS for wavefront generation and performs the traceback of traveling wavefronts to find the shortest paths.

Figures 2 (a)–(c) show a wave propagation in two-dimensional DRDS with the size of 128×128. In Fig. 2 (d), 26 snapshots of wavefronts at 100-step time intervals are superimposed to form a composite image. Each wavefront represents a set of equidistant locations from the starting point at each time step, and hence we can derive the shortest path by tracing back the history of wavefront position from the goal to the starting point.

The proposed algorithm consists of two operations: *Forward Operation* and *Backward Operation*. The Forward Operation is to generate a traveling wave in the excitable DRDS and record snapshots of equidistant wave

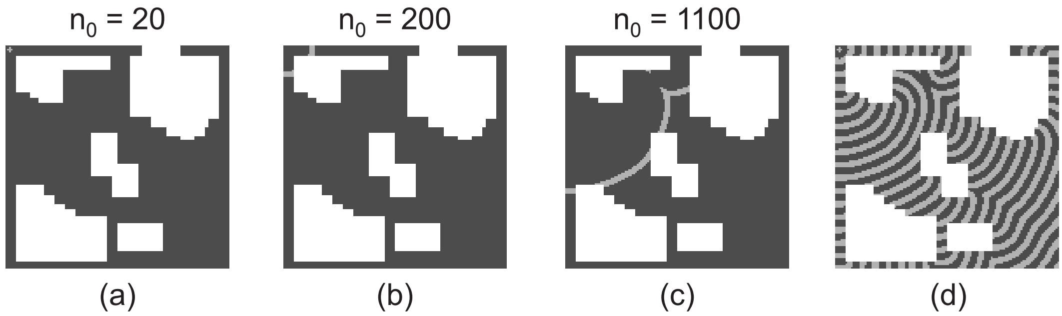

Figure 2. Wave propagation in two-dimensional DRDS with obstacles: (a)–(c) wave propagation, (d) superposition of traveling waves taken every 100-step.

procedure *Forward Operation*
Input
a map (obstacle information), a starting point (n_1^S, n_2^S), a goal (n_1^G, n_2^G);
Output
$W(n_0)$: a list of points (n_1, n_2) in two-dimensional space at which the value of $x_1(n_0, n_1, n_2)$ is higher than a specific threshold value Thr (that is, $W(n_0)$ stores the list of coordinates at which the traveling wave exists),
n_0^G: the time step when the traveling wave arrives at the goal (n_1^G, n_2^G);

begin
Set a high concentration (that is higher than the threshold Thr) to $x_1(0, n_1^S, n_2^S)$;
Set 0 to $x_1(0, n_1, n_2)$ for the coordinates $(n_1, n_2) \neq (n_1^S, n_2^S)$
Set 0 to $x_2(0, n_1, n_2)$ for all the coordinates (n_1, n_2);
Store (n_1^S, n_2^S) in $W(0)$;
$n_0 := 0$; { Initialize the time step }
while the traveling wave does not arrive at (n_1^G, n_2^G) **do**
begin
Compute the excitable DRDS (2) for one step assuming the boundary condition defined by the map, and derive $x_1(n_0+1, n_1, n_2)$ and $x_2(n_0+1, n_1, n_2)$;
Store the coordinates (n_1, n_2) of the wavefronts into $W(n_0+1)$ (i.e., the coordinates (n_1, n_2) at which the value $x_1(n_0+1, n_1, n_2)$ is higher than Thr);
$n_0 := n_0 + 1$
end;
$n_0^G := n_0$
end.

Figure 3. Algorithm for *Forward Operation.*

patterns at specific time intervals. Backward Operation, on the other hand, is to trace back the wavefronts from the goal to starting point to find the optimal pathways. Figure 3 and 4 show the algorithms for the Forward and Backward Operations, respectively. The proposed algorithm organizes the Forward and Backward Operations so that they require only four inputs: a map, a starting point (n_1^S, n_2^S), a goal (n_1^G, n_2^G) and a time step resolution Δ for traceback.

Figure 5 shows typical examples of shortest path search in complicated mazes, where multiple goals are specified in advance. We can observe that all the obtained paths avoid obstacles and are the shortest paths, in terms of Euclidean distance, for the given goals.

4. Conclusion

This paper presents a shortest path search algorithm in a two-dimensional space using the excitable Digital Reaction-Diffusion System (DRDS). The traveling wave generated by the excitable DRDS has two significant features: (i) propagation with constant velocities and (ii) annihilation in collisions with other waves. These features are effectively used to find the shortest paths in mazes with various obstacles. The proposed algorithm could be applied to various navigation tasks defined in two-dimensional space, and could also be extended to shortest path search algorithms for higher-dimensional space.

References

[1] A. M. Turing, "The chemical basis of morphogenesis," *Phil. Trans. Roy. Soc.*, vol. B237, pp. 37–72, Aug. 1952.

[2] J. D. Murray, *Mathematical Biology*, Springer-Verlag, Berlin, 1993.

[3] S. Kondo and R. Asai, "A reaction-diffusion wave on the skin of the marine angelfish pomacanthus," *Nature*, vol. 376, pp. 765–768, Aug. 1995.

[4] A. S. Sherstinsky and R. W. Picard, "M-lattice: from morphogenesis to image processing," *IEEE Trans. Image Processing*, vol. 5, no. 7, pp. 1137–1150, July 1996.

[5] K. R. Crounse and L. O. Chua, "Methods for image processing and pattern formation in cellular neural networks: a tutorial," *IEEE Trans. Circuits Syst.-I*, vol. 42, no. 10, pp. 583–601, Oct. 1995.

```
procedure Backward Operation
  Input
    (n_1^S, n_2^S), (n_1^G, n_2^G), W(n_0), n_0^G,
    Δ: a resolution of the time step interval for traceback operation;
  Output
    Path: a list of points on the shortest path from (n_1^S, n_2^S) to (n_1^G, n_2^G);

begin
  Set (n_1^G, n_2^G) to Path and a search list;
  n_0 := n_0^G; { initialize the time-step counter n_0 to the time step when the wave arrives at (n_1^G, n_2^G) }
  while the search list is not empty do
    begin
      Get a search point (n_1^b, n_2^b) from the search list and removes it from the list;
      while n_0 > 0 do
        begin
          Get (n_1, n_2) of W(n_0−Δ) with the shortest distance from (n_1^b, n_2^b);
          if (n_1, n_2) is one then
            Set (n_1, n_2) to (n_1^b, n_2^b);
          else
            begin
              if the pathway branches at time step n_0−Δ then
                begin
                  Set one of (n_1, n_2) to (n_1^b, n_2^b);
                  Store other points of (n_1, n_2) into the search list
                end;
              else
                Set the average of (n_1, n_2) to (n_1^b, n_2^b);
            end;
          Store (n_1^b, n_2^b) into Path;
          n_0 := n_0 − Δ { update the time step }
        end;
      Store (n_1^S, n_2^S) into Path
    end;
  Display Path on the map
end.
```

Figure 4. Algorithm for *Backward Operation*.

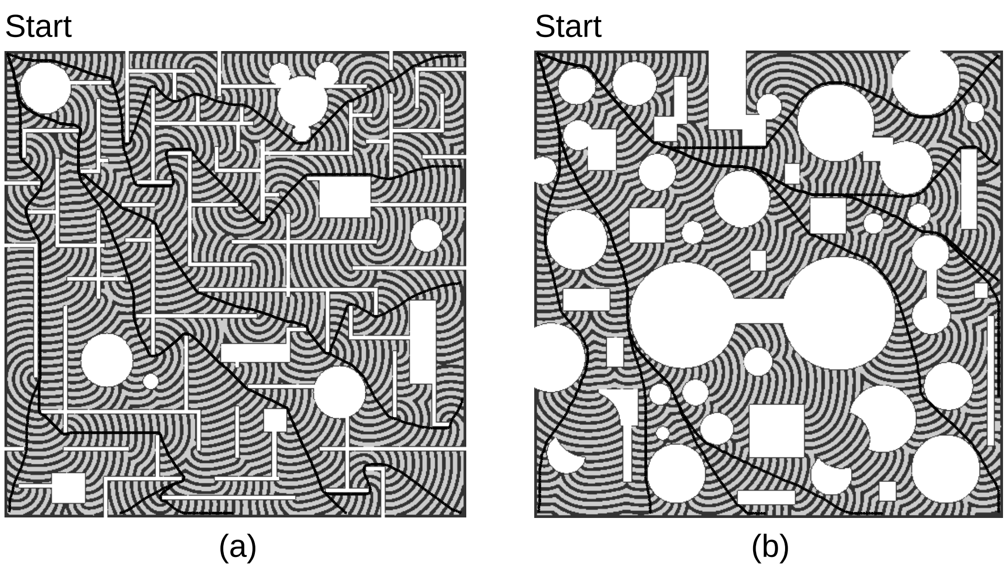

Figure 5. Shortest path search on 512×512 space with convex (a) and non-convex (b) obstacles.

[6] K. Ito, T. Aoki, and T. Higuchi, "Digital reaction-diffusion system —A foundation of bio-inspired texture image processing—," *IEICE Trans. Fundamentals*, vol. E84-A, no. 8, pp. 1909–1918, Aug. 2001.

[7] K. Ito, T. Aoki, and T. Higuchi, "Fingerprint restoration using digital reaction-diffusion system and its evaluation," *IEICE Trans. Fundamentals*, vol. E86-A, no. 8, pp. 1916–1924, Aug. 2003.

[8] O. Steinbock, Á. Tóth, and K. Showalter, "Navigating complex labyrinths: optimal paths from chemical waves," *Science*, vol. 267, pp. 868–871, Feb. 1995.