# RECONSTRUCTING OCCLUDED REGIONS USING FAST WEIGHTED PCA 

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#### Abstract

Reconstructing occluded regions of the object is to automatically detect the occluded regions and background in the image and reconstruct these regions using image interpolation. This paper proposes a novel occluded region reconstruction method using Fast Weighted Principal Component Analysis (FW-PCA). The computation time of the weighted PCA can be reduced by using only the effective regions when calculating the principal component scores. The occluded regions are accurately detected by recursively updating the weight for each pixel in the image using FW-PCA. Then, the occluded regions can be reconstructed using the final weight. Thorough a set of experiments, we demonstrate that the proposed method exhibits higher performance than the conventional method.


Index Terms- image interpolation, occluded region reconstruction, principal component analysis, eigenspace

## 1. INTRODUCTION

Reconstructing occluded regions of the object is to automatically detect the occluded regions, and reconstruct these regions using image interpolation. The occluded region reconstruction is one of the essential techniques in the field of factory automation, person identification, image editing, image recognition, etc. The most of conventional methods for reconstructing occluded regions need to manually define the occluded regions, since these methods focus on the reconstruction accuracy. On the other hand, this paper focuses on the automatic and accurate reconstruction of the occluded regions of the object.

So far, there are some works on the automatic reconstruction of occluded regions of the object [1, 2, 3, 4]. In face image processing, Refs. [1, 2] have proposed occluded region reconstruction methods using Active Appearance Model (AAM) [5]. These methods can extract the face region and remove the occluded objects such as an eyeglass, a scarf, etc. using AAM. If these methods are applied to other classes, we need to create a model for the new target. The PCA (Principal Component Analysis)-based methods proposed in Refs. [3, 4] can be applied to reconstruct occluded regions of generic objects. Leonardis, et al. [3] have proposed a reconstruction
method using the weighted PCA. This method estimates the optimal hypothesis, i.e., object region, using the weighted PCA with a set of subsampled points in the input image and reconstructs the occluded regions based on the estimated hypothesis. Storer, et al. [4] have proposed the Fast-Robust PCA (FR-PCA) to reconstruct the occluded regions. This method reconstructs the occluded regions using the normal PCA with the eigenvectors generated from a set of subsampled input images in advance. The accuracy and computation time of the above PCA-based methods depend on the number of subsampling patterns required for generating the eigenspace.

On the other hand, this paper proposes a novel fast and accurate occluded region reconstruction method using the Fast Weighted PCA (FW-PCA). FW-PCA calculates the principal component scores using only effective pixels in the image to detect occluded regions. The occluded regions are accurately detected by recursively updating the occluded regions using FW-PCA. Then, the occluded regions can be reconstructed using the eigenspace generated from the training set. Using the proposed method, we cannot only reconstruct but also detect the occluded regions. Through a set of experiments using images in Amsterdam Library of Object Images (ALOI) [6], we demonstrate that the proposed method exhibits efficient performance compared with the conventional algorithm.

## 2. RELATED WORKS

This section describes the occluded region reconstruction methods using PCA and FR-PCA.

### 2.1. Reconstructing Image Using PCA

We briefly introduce the image reconstruction method using PCA. Let $x_{l}$ be an N-dimensional vector consisting of pixel values $x_{l 1}, x_{l 2}, \cdots, x_{l N}$ of the image $\boldsymbol{I}_{l}$ as $\boldsymbol{x}_{l}=$ $\left[\begin{array}{llll}x_{l 1} & x_{l 2} & \cdots & x_{l N}\end{array}\right]^{T}$. Let $\boldsymbol{X}=\left[\begin{array}{llll}\boldsymbol{x}_{1} & \boldsymbol{x}_{2} & \cdots & \boldsymbol{x}_{M}\end{array}\right]$ be the learning sample, where $M$ is the number of images. We calculate the covariance matrix between pixels from $\boldsymbol{X}$ and obtain the
eigenspace $\boldsymbol{E}$ as

$$
\begin{align*}
\boldsymbol{E} & =\left[\begin{array}{cccc}
e_{11} & e_{21} & \cdots & e_{D 1} \\
e_{12} & e_{22} & \cdots & e_{D 2} \\
\vdots & \vdots & \ddots & \vdots \\
e_{1 N} & e_{2 N} & \cdots & e_{D N}
\end{array}\right] \\
& =\left[\begin{array}{llll}
\boldsymbol{e}_{1} & e_{2} & \cdots & \boldsymbol{e}_{D}
\end{array}\right] \tag{1}
\end{align*}
$$

where $D(D \leq M)$ is the number of eigenvectors (it also means the dimension of the eigenspace).

As the eigenspace $\boldsymbol{E}$ is calculated by PCA, the reconstructed image $\hat{\boldsymbol{x}}$ is estimated by

$$
\begin{equation*}
\hat{\boldsymbol{x}} \simeq \boldsymbol{E} \boldsymbol{p} \quad\left(\Leftarrow \hat{x}_{i} \simeq \sum_{d=1}^{D} e_{d i} p_{d}\right) \tag{2}
\end{equation*}
$$

where $\boldsymbol{p}$ is the principal component score, and $i=1 \sim$ $N . \boldsymbol{p}$ is calculated by projecting the input image $\boldsymbol{x}$ onto the eigenspace $\boldsymbol{E}$ as

$$
\begin{equation*}
\boldsymbol{p}=\boldsymbol{E}^{T} \boldsymbol{x} \quad\left(\Leftarrow p_{d}=\sum_{i=1}^{N} e_{d i} x_{i}\right) . \tag{3}
\end{equation*}
$$

In the case that the missing pixels such as occlusion included in the input image, the reliability of the principal component score $p$ is decreased. So, the reconstructed image is also not reliable. To address this problem, Refs. [3, 7] have proposed the reconstruction methods where the principal component score $\boldsymbol{p}$ is calculated only from a set of the effective pixels in the input image. In these methods, the reconstruction error $C(\boldsymbol{p})$ in a set of effective pixels is defined by

$$
\begin{equation*}
C(\boldsymbol{p})=\sum_{i=1}^{N} w_{i}\left(x_{i}-\hat{x}_{i}\right)^{2}=\sum_{i=1}^{N} w_{i}\left(x_{i}-\sum_{d=1}^{D} e_{d i} p_{d}\right)^{2} \tag{4}
\end{equation*}
$$

The principal component score $\hat{\boldsymbol{p}}$ having minimum reconstruction error $C(\boldsymbol{p})$ is estimated by using the least-square method as follows

$$
\begin{equation*}
\hat{\boldsymbol{p}} \simeq\left(\boldsymbol{E}^{T} \boldsymbol{W} \boldsymbol{E}\right)^{-1} \boldsymbol{E}^{T} \boldsymbol{x} \tag{5}
\end{equation*}
$$

where $\boldsymbol{W}=\operatorname{diag}(\boldsymbol{w})$ is a diagonal matrix whose diagonal elements are weights $\boldsymbol{w}=\left[\begin{array}{llll}w_{1} & w_{2} & \cdots & w_{N}\end{array}\right]$. Each element $w_{i}$ is assigned 1 for the valid pixel or 0 for the missing pixel. If the missing pixels in the input image are selected in advance, we can reconstruct them using the eigenspace of PCA.

### 2.2. Reconstructing Occluded Regions Using FR-PCA

We briefly explain the reconstruction method using FR-PCA [4]. In this method, the occluded regions are detected by the following two steps. In the first step, the occluded pixels are detected based on the reconstruction errors of eigenspaces
generated from the training image which are randomly sampled. In the second step, the whole image is reconstructed using the estimated effective pixels. Note that these steps are iterated until the error between the input and reconstructed images is below the threshold. The computation time of this method is relatively short, since the small data sets are used to perform PCA. On the other hand, this method requires a lot of patterns for random sampling of the training images, since the reconstruction accuracy of this method depends on the number of subsampling patterns. Also, this method uses a part of effective pixels to reduce the computation time. So, the whole occluded regions cannot be detected.

## 3. PROPOSED METHOD

In thie section, we describe the proposed method which can detect and reconstruct the occluded regions in the input image. Unlike the conventional methods [3, 4], the proposed method do not need any initial patterns to detect the occluded regions, and detect the whole occluded regions. The proposed method consists of 2 key components: (i) FW-PCA and (ii) the occluded reconstruction algorithm. In the following, we explain the details of the 2 key components.

### 3.1. FW-PCA

FW-PCA approximately calculates the correlation coefficient between the input image and the eigenvectors and the amplitude of the input image to reduce the computation time.

The $d$-th principal component score $p_{d}$ can be calculated as the inner product of the input image $\boldsymbol{x}$ and the $d$-th eigenvector $e_{d}$ as follows

$$
\begin{equation*}
p_{d}=\sum_{i=1}^{N} e_{d i} x_{i}=\left\|\boldsymbol{e}_{d}\right\|\|\boldsymbol{x}\| \cos \theta_{d} \tag{6}
\end{equation*}
$$

where $\|x\|$ and $\left\|e_{d}\right\|$ indicate the amplitude of the input image and the $d$-th eigenvector, respectively, and $\cos \theta_{d}$ indicates the correlation coefficient between the input image and the $d$-th eigenvector.

If the input image has occluded pixels, the correlation coefficient $\cos \theta_{d}$ and the amplitude $\|\boldsymbol{x}\|$ cannot be directly calculated. So, the correlation coefficient $\cos \theta_{d}$ and the amplitude $\|x\|$ are approximately calculated by

$$
\begin{align*}
\cos \hat{\theta}_{d} & =\frac{\sum_{i=1}^{N} w_{i} e_{d i} x_{i}}{\sqrt{\sum_{i=1}^{N} w_{i} x_{i}^{2}} \sqrt{\sum_{i=1}^{N} w_{i} e_{d i}^{2}}},  \tag{7}\\
\|\hat{\boldsymbol{x}}\| & =\frac{\sqrt{\sum_{i=1}^{N} e_{d i}^{2}} \sqrt{\sum_{i=1}^{N} w_{i} x_{i}^{2}}}{\sqrt{\sum_{i=1}^{N} w_{i} e_{d i}^{2}}} \tag{8}
\end{align*}
$$

Replacing the correlation coefficient and the amplitude of the input image with Eq. (7) and Eq. (8), respectively, Eq. (6)
can be rewritten as

$$
\begin{equation*}
\hat{p}_{d}=\left\|\boldsymbol{e}_{d}\right\|\|\hat{\boldsymbol{x}}\| \cos \hat{\theta}_{d}=\frac{\sum_{i=1}^{N} e_{d i}^{2} \sum_{i=1}^{N} w_{i} e_{d i} x_{i}}{\sum_{i=1}^{N} w_{i} e_{d i}^{2}} \tag{9}
\end{equation*}
$$

Note that Eq. (9) is equivalent to Eq. (5) if the input image has no missing pixel, while Eq. (9) is different from Eq. (5) if the input image has missing pixels. The use of FW-PCA makes it possible to quickly calculate the weighted PCA for the input image having the missing pixels.

### 3.2. Occluded Region Reconstruction Algorithm

Fig. 1 shows the flow diagram of the proposed reconstruction algorithm.

### 3.2.1. Learning Step

This step is to generate the eigenspaces of the training images $\boldsymbol{X}=\left[\begin{array}{llll}\boldsymbol{x}_{1} & \boldsymbol{x}_{2} & \cdots & \boldsymbol{x}_{M}\end{array}\right]$ in the same way in Sect. 2.1, where the target object is taken under the uniform background.

### 3.2.2. Reconstruction Step

This step is to reconstruct the occluded regions of the input image using FW-PCA.

The occluded regions are updated so as to minimize the reconstruction error in the effective pixels by iteratively calculating FW-PCA. Finally, we obtain the reconstructed image and the detected occluded regions. The detailed procedure is the following.

Step 1: Initialize the weight $\boldsymbol{w}$ to 1.
Step 2: Obtain the reconstructed image $\hat{\boldsymbol{x}}$ by calculating FWPCA (Eq. (9)) using the input image $\boldsymbol{x}$ and the weight $w$ 。

Step 3: Update the weight $\boldsymbol{w}$ by comparing the reconstructed image $\hat{\boldsymbol{x}}$ and the input image $\boldsymbol{x}$ as follows

$$
w_{i}= \begin{cases}1 & \text { if }\left|\hat{x}_{i}-x_{i}\right|<\theta_{x} \\ 0 & \text { otherwise }\end{cases}
$$

Step 4: Repeat Step 2~3 at $T$ times.
The final outputs of $\hat{\boldsymbol{x}}$ and $\boldsymbol{w}$ are the reconstruction result and the occluded region, respectively.

## 4. EXPERIMENTS AND DISCUSSION

We evaluate the reconstruction accuracy and the computation time of the proposed method using the 3D object images in Amsterdam Library of Object Images (ALOI) [6] and hand images.

The ALOI database consists of 72,000 images of 1,000 objects with the image size $192 \times 144$ pixels. Each object


Fig. 1. Flow diagram of the proposed method.


Fig. 2. Examples of the images used in the experiments.
is placed on the turntable. The turntable is rotated through 360 degrees, and 72 images are taken per object. In this experiment, we use 720 images of 10 objects from the ALOI database. The hand images are taken by changing the shape of the left hand on the black background. We use 72 hand images with the image size $192 \times 144$ pixels. Thus, we use the sample set consisting of 792 images of 11 classes ( 72 im ages per class) as shown in Fig. 2. The images in Class0 to Class9 are from the ALOI database and the images in Class10 are from the hand images. The sample images are classified into training images and test images. The sample images labeled even frame number are classified into training images, which are used for generating the eigenspace of each class. The sample images labeled odd frame number are classified into test images, which are used for evaluating the reconstruction performance. As for the test images, we add 4 occlusion patterns and 5 background patterns for each image, so the total number of patterns is 20 and the total number of the test images is 7,920 ( 36 frames $\times 20$ patterns $\times 11$ classes).

In this experiment, we compare the reconstruction accuracy and the computation time of the conventional method using FR-PCA [4] and the proposed method, where both methods are implemented using C++. We use $D=32$ dimensions for both methods. The parameter set for FR-PCA is the same as Ref. [4]. The number of subsampled pixels is $1 \%$ of the whole pixels and the number of the small subspaces is $N_{s}=500$ and 1,000 . The parameter set for the proposed method is $\theta_{x}=16, T=1,2,4$ and 8 . Note that the proposed method with $T=1$ is equivalent to the normal PCA. The reconstruction accuracy is evaluated by Root Mean Square


Fig. 3. Reconstruction images (upper) and the occluded regions (lower) obtained by using the proposed method with the different $T$.

Table 1. Summary of the computation time and the average RMS errors.

| Method | Computation time <br> [msec.] | RMS error <br> [pixel] |
| :---: | :---: | :---: |
| FR-PCA $\left(N_{s}=500\right)$ | 35.73 | 21.44 |
| FR-PCA $\left(N_{s}=1000\right)$ | 60.29 | 21.25 |
| Proposed $(T=1)$ | 13.37 | 22.16 |
| Proposed $(T=2)$ | 21.58 | 19.60 |
| Proposed $(T=4)$ | 38.66 | 18.84 |
| Proposed $(T=8)$ | 72.16 | 18.57 |

(RMS) errors between the original and reconstructed images. The computation time is evaluated by using Intel Core2 Duo ( 2.2 GHz ) processor.

Fig. 3 shows the reconstructed images and the occluded regions obtained by using the proposed method with the different $T$. As a result, the reconstruction accuracy is enhanced by iteratively calculating FW-PCA. Also, we observe that the accuracy of the proposed method converges for $T \geq 8$.

Fig. 4 shows the experimental results of FR-PCA and the proposed method. As a result, the proposed method can reconstruct the occluded regions better than FR-PCA. FR-PCA estimates only a part of effective pixels, while the proposed method estimates all the effective pixels and iteratively updates the reconstruction results while evaluating the reconstruction error.

Table 1 shows the summary of the experimental results. The computation time of both methods depends on the parameter set. Focusing on the comparable computation time, the average RMS error of the proposed method is smaller than that of FR-PCA.

As is observed in the above experiments, the proposed method using FW-PCA exhibits efficient performance for reconstructing the occluded regions in the input image compared with the conventional method using FR-PCA.


Fig. 4. Reconstruction results: (a) input images, (b) results of FR-PCA, (c) results of the proposed method $(T=1)$, (d) result of the proposed method $(T=8)$ and (e) original images.

## 5. CONCLUSION

This paper has proposed a novel occluded region reconstruction method using Fast Weighted Principal Component Analysis (FW-PCA). Thorough a set of experiments, we have demonstrated that the proposed method exhibits higher performance than the conventional method. In future work, we will apply the proposed method to detect the occluded regions of face images in order to enhance the face recognition algorithms.

## 6. REFERENCES

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