# A Geometric Correction Method Using Stereo Vision for Projected Images

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***Abstract*—This paper proposes a geometric correction method using stereo camera for projected images. When we use a projector in the various places, a projected image is deformed due to the shape of a display surface, the positional relationship between a projector and a display surface, etc. Addressing this problem, we propose a geometric correction method for projected images, which employs 3D measurement technique with the stereo camera. The proposed method enables us to automatically correct the deformation of projected images during the projection of ordinary images. In our method, we employ the correspondence search technique based on Phase-Only Correlation (POC) to obtain dense 3D shape of the display surface. A set of experiments demonstrates that the proposed method is effective for correcting deformation of projected images.**

***Index Terms*—projector-camera system, stereo vision, geometric correction, phase-only correlation,SIFT**

## I. Introduction

Recently, a projector-camera system has been proposed to combine projection technology with computer vision [1]. One of the applications is to project desired images which have no geometric deformation even if the shape of surface is not plane. In this application, we need the high-accuracy correspondence between the projector images and the camera images.

So far, there are two approaches for geometric correction of complex display surfaces. The first approach using per-pixel mapping between projector and camera images [2] provides adequate precision for geometric correction, but does not support arbitrary view. The per-pixel mapping is performed by projecting the special patterns, e.g., structured light patterns, on a display surface. Therefore, when changing the environment, e.g., the screen is moved, the correspondence between projector and camera images must be updated by reprojecting the special patterns. The second approach using a 3D model of the display surface [3] supports moving users, but requires a high-accuracy 3D model of the display surface. Although the 3D measurement techniques using projectors and cameras have been proposed [1], the special pattern projection on a display surface is also required as well as the first approach. Therefore, if the camera and projector are moved or the shape of the display surface is changed, we need to stop showing the contents and to project special pattens again.

On the other hand, we propose an automatic geometric correction method using stereo camera for projected images. In the proposed method, we correct geometric distortion of projected images using not the special light patterns, but ordinary images. The proposed method only needs the calibrated stereo camera. Using the stereo camera, we can obtain both the perspective projection matrix of the projector and 3D shape of the surface during projection of ordinary images. To achieve this, we employ two image matching techniques: (i) SIFT (Scale-Invariant Feature Transform) [4] and (ii) POC (Phase-Only Correlation) [5], [6], [7], [8]. SIFT is for estimating the perspective projection matrix of the projector and POC is for obtaining the high-accuracy and dense 3D shape of the display surface. Hence, we can correct projected images immediately when changing the environment. Through a set of experiments, we demonstrate that the proposed method is effective for correcting distortion of projected images.

## II. Phase-Only Correlation

We introduce the principle of a Phase-Only Correlation (POC) function (which is sometimes called the "phase-correlation function") [5], [6], [7], [8]. The POC function is defined as the inverse Discrete Fourier Transform (DFT) of normalized cross-power spectrum. When two images are similar, their POC function gives a distinct sharp peak. When two images are not similar, the peak drops significantly. The height of the peak gives a good similarity measure for image matching, and the location of the peak shows the translational displacement between the images.

Takita, et al., [7] have proposed an efficient method of sub-pixel correspondence matching, which employs (i) a coarse-to-fine strategy using image pyramids for robust correspondence search and (ii) a POC-based sub-pixel image matching technique for finding a pair of corresponding points with sub-pixel displacement accuracy. Let $\boldsymbol{p}$ be a coordinate vector of a reference pixel in the reference $I(n_1, n_2)$. The problem of sub-pixel correspondence search is to find a real-number coordinate vector $\boldsymbol{q}$ in the input image $J(n_1, n_2)$ that corresponds to the reference pixel $\boldsymbol{p}$ in $I(n_1, n_2)$. We briefly explain the procedure as follows.

**Step 1**: For $l = 1, 2, \cdots, l_{max}$, create the $l$-th layer images $I_l(n_1, n_2)$ and $J_l(n_1, n_2)$, i.e., coarser versions of $I_0(n_1, n_2)$

and $J_0(n_1, n_2)$, recursively as follows:

$$I_l(n_1, n_2) = \frac{1}{4}\sum_{i_1=0}^{1}\sum_{i_2=0}^{1} I_{l-1}(2n_1 + i_1, 2n_2 + i_2),$$
$$J_l(n_1, n_2) = \frac{1}{4}\sum_{i_1=0}^{1}\sum_{i_2=0}^{1} J_{l-1}(2n_1 + i_1, 2n_2 + i_2).$$

In this paper, we employ $l_{max} = 3$.

**Step 2**: For every layer $l = 1, 2, \cdots, l_{max}$, calculate the coordinate $\boldsymbol{p}_l = (p_{l1}, p_{l2})$ corresponding to the original reference point $\boldsymbol{p}_0$ recursively as follows:

$$\boldsymbol{p}_l = \lfloor \tfrac{1}{2}\boldsymbol{p}_{l-1} \rfloor = (\lfloor \tfrac{1}{2} p_{l-1\ 1} \rfloor, \lfloor \tfrac{1}{2} p_{l-1\ 2} \rfloor), \quad (1)$$

where $\lfloor z \rfloor$ denotes the operation to round the element of $z$ to the nearest integer towards minus infinity.

**Step 3**: We assume that $\boldsymbol{q}_{l_{max}} = \boldsymbol{p}_{l_{max}}$ in the coarsest layer. Let $l = l_{max} - 1$.

**Step 4**: From the $l$-th layer images $I_l(n_1, n_2)$ and $J_l(n_1, n_2)$, extract two sub-images (or image blocks) $f_l(n_1, n_2)$ and $g_l(n_1, n_2)$ with their centers on $\boldsymbol{p}_l$ and $2\boldsymbol{q}_{l+1}$, respectively. The size of image blocks is $W \times W$ pixels. In this paper, we employ $W = 32$.

**Step 5**: Estimate the displacement between $f_l(n_1, n_2)$ and $g_l(n_1, n_2)$ with pixel accuracy using POC-based image matching. Let the estimated displacement vector be $\boldsymbol{\delta}_l$. The $l$-th layer correspondence $\boldsymbol{q}_l$ is determined as follows:

$$\boldsymbol{q}_l = 2\boldsymbol{q}_{l+1} + \boldsymbol{\delta}_l. \quad (2)$$

**Step 6**: Decrement the counter by 1 as $l = l - 1$ and repeat from Step 4 to Step 6 while $l \geq 0$.

**Step 7**: From the original images $I_0(n_1, n_2)$ and $J_0(n_1, n_2)$, extract two image blocks with their centers on $\boldsymbol{p}_0$ and $\boldsymbol{q}_0$, respectively. Estimate the displacement between the two blocks with sub-pixel accuracy using POC-based image matching. Let the estimated displacement vector with sub-pixel accuracy be denoted by $\boldsymbol{\delta} = (\delta_1, \delta_2)$. Update the corresponding point as

$$\boldsymbol{q} = \boldsymbol{q}_0 + \boldsymbol{\delta}. \quad (3)$$

In the case of parallel stereo, we can employ 1D POC matching to obtain high-accuracy and dense corresponding pairs [8]. The use of 1D POC matching makes it possible to reduce computation time without sacrificing the accuracy of corresponding points. In this paper, we employ 1D POC-based correspondence matching to reconstruct the 3D shape of the display surface.

## III. Geometric Correction Method

In this section, we present the proposed geometric correction method for projected images. The geometric correction procedure consists of 3 steps: (i) calibration of the projector, (ii) 3D measurement of the display surface and (iii) projector image transformation. At first, we estimate the perspective projection matrix of the projector using SIFT-based feature point matching and POC-based correspondence matching. Then, we measure the 3D surface of the object with the stereo camera pair using 1D POC-based correspondence matching. Finally, using the result of 3D measurement, we correct the deformation of projected images. We describe the details of each step as follows (Fig. 1):

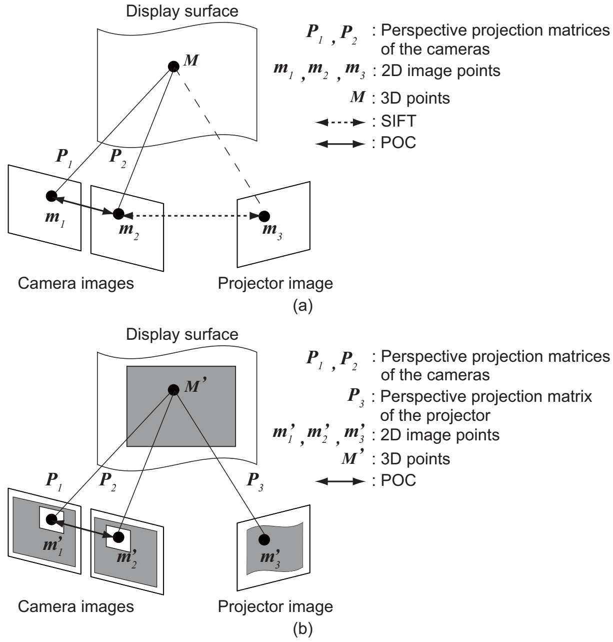


Fig. 1. Overview of the proposed method: (a) calibration of the projector, and (b) 3D measurement of the display surface and projector image transformation.

**(i) Calibration of the projector**

Projector calibration is performed by obtaining the perspective projective matrix $\boldsymbol{P}_3$ which is required to correct deformation of the projected images using the results of 3D measurement of the display surface. $\boldsymbol{P}_3$ has only 11 degrees of freedom and can be estimated from more than six 3D points and their coordinates on the projector image and geometric relationship among projector, stereo camera and display surface. If the projector calibration can be performed using the object with known geometry in advance, we can estimate $\boldsymbol{P}_3$. In this case, there are two major problems as follows:

- The zoom and focus setting of the projector should be fixed, since the intrinsic parameter of the projector depends on the zoom and focus setting.
- The positional relationship between the projector and the stereo camera should be fixed, since the extrinsic parameter of the projector depends on the positional relationship.

Addressing the above problems, we propose the estimation method of the perspective projection matrix $\boldsymbol{P}_3$ of the projector during projecting ordinary images by using SIFT to obtain the correspondence between the projector and the camera, and POC to obtain the correspondence between the stereo camera images.

We describe the details of the proposed method as follows. At first, we obtain the corresponding points between one of

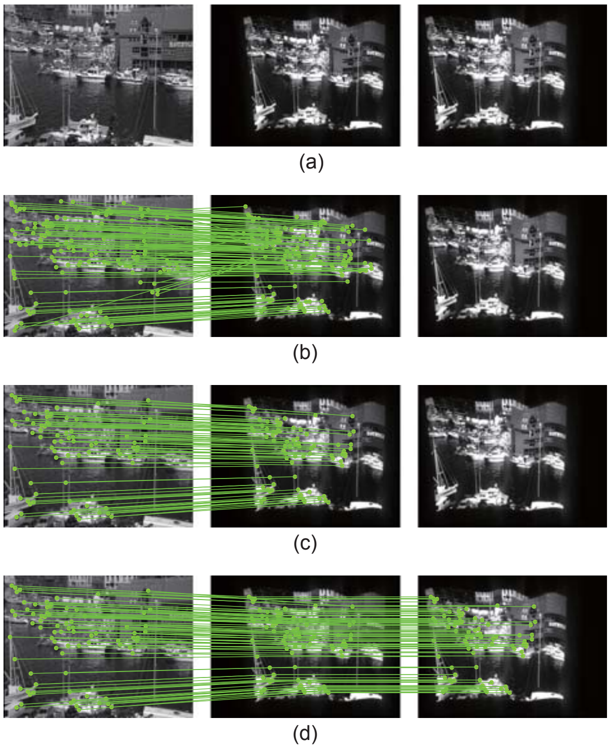

Fig. 2. Correspondence search among projector and stereo images: (a) projector image (left) and stereo image (right), (b) result of SIFT-based feature point matching, (c) result of the outlier removal using RANSAC and (d) result of POC-based correspondence search.

camera images and projector image using SIFT-based feature point matching as shown in Fig. 2 (b). Since the camera image has nonlinear deformation due to the shape of the display surface, the results of SIFT matching include outliers. To estimate the perspective projection matrix $\boldsymbol{P}_3$ of the projector with high accuracy, we need to eliminate outliers. So, we employ RANSAC (RANdom SAmple Consensus) [9] to obtain correct corresponding point pairs between camera and projector images. Fig. 2 (c) shows the corresponding point pairs between camera and projector images after eliminating outliers.

Next, we obtain corresponding points between stereo camera image using POC-based correspondence matching. Note that the reference points for POC-based correspondence matching should be the same as the keypoints obtained from SIFT-based feature point matching. Finally, we can obtain correspondence among projector and stereo camera images as shown in Fig. 2 (d). We can calculate the 3D coordinates of the feature points, since the stereo camera is calibrated in advance. As a result, the 2D coordinates of the projector image correspond to 3D coordinates of the projector image. The perspective projection matrix $\boldsymbol{P}_3$ of the projector can be estimated from the obtained correspondence using the least-square estimation method.

Finally, we perform bundle adjustment [10] in order to obtain optimal $\boldsymbol{P}_3$ by minimizing the reprojection error $C$ defined by

$$C = \sum_{i=1}^{n} \sum_{j=1}^{3} \left( \| u_j^i - u_j^{\hat{i}} \|^2 + \| v_j^i - v_j^{\hat{i}} \|^2 \right), \qquad (4)$$

where $(u_j^i, v_j^i)$ denote $i$-th corresponding points and $(u_j^{\hat{i}}, v_j^{\hat{i}})$ denote the coordinates reprojected from $i$-the 3D coordinates with parameters of cameras and projector. $j$ $(= 1, 2, 3)$ indicates left and right cameras and projector, respectively. $n$ denotes the total number of corresponding points.

**(ii) 3D measurement of the display surface**

We perform high-accuracy and dense 3D measurement of the display surface using phase-based correspondence matching in order to correct geometric deformation of the projected image during projecting ordinary images.

At first, the stereo camera image is rectified using the result of camera calibration as shown in Fig. 3 (a), where the stereo camera is calibrated in advance. Next, we set the reference points on the left camera image in a reticular pattern and find corresponding points on the right camera image using 1D POC-based correspondence matching. In the case of ordinary images, there are some areas having poor texture. These areas are hard to find accurate correspondence and resulted in outliers. Addressing this problem, we evaluate the accuracy of corresponding points by using peak value of 1D POC. If the peak value is below the threshold, the corresponding point is determined as an outlier. Fig. 3 (b) shows corresponding point pairs after outlier removal.

To obtain dense correspondence, we estimate approximated corresponding points for all outliers. we focus on the 8-neighbor corresponding pints of each outlier and calculate disparities only for precise corresponding points. The approximated coordinate for outlier is obtained by adding the average disparity to the coordinate of the reference point. Fig. 3 (c) shows corresponding point pairs after re-estimation of outliers.

**(iii) Projector image transformation**

The real-world 3D coordinates are transformed to the projector image coordinates using the perspective projection matrix of the projector $\boldsymbol{P}_3$. We create mesh from the points on the projector image $\boldsymbol{m}_3^{'}$, which are obtained by $\boldsymbol{P}_3$ and $\boldsymbol{M}^{'}$. Then, the original images or video sequences to be projected are transformed based on the mesh. Projecting this image, we can observe a deformation-corrected image.

## IV. Experiments and Discussion

This section describes experimental evaluation of the proposed method. Fig. 4 shows our experimental environment. We use "Cannon X700" with a resolution of $1,024 \times 768$ pixels and the "Point Grey SCOR-14SOC" with $1,280 \times 960$ pixels. The display surface used in the experiment is a white curtain as shown in Fig. 4. The baseline of the stereo camera is 10cm. The length between projector and display surface is about 80cm. The length between stereo camera and display surface is about 115cm. The specifications of PC is shown in Table I. We use a 25mm×25mm checkerboard pattern and MATLAB Camera Calibration Toolbox [11] to calibrate the stereo camera

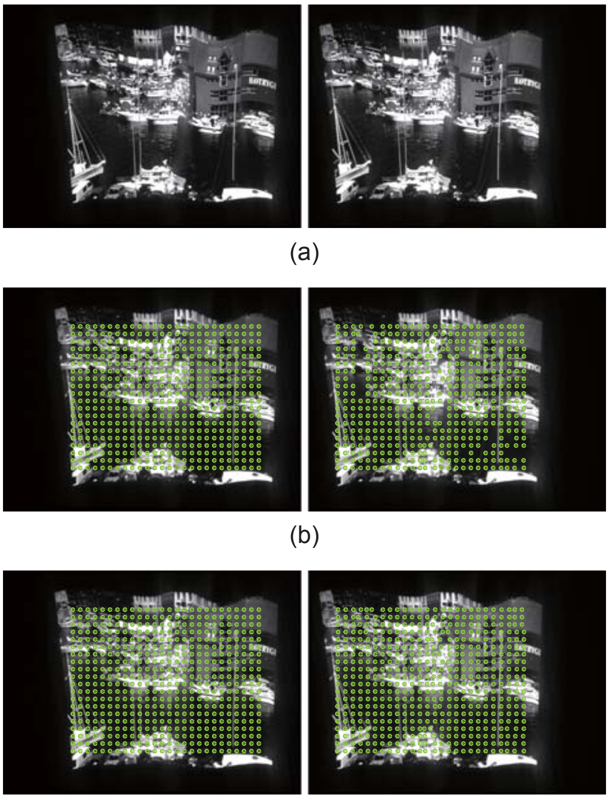

Fig. 3. Correspondence search between stereo images: (a) stereo camera images after rectification, (b) corresponding points obtained by the POC-based correspondence search, and (c) corresponding points after re-estimation.

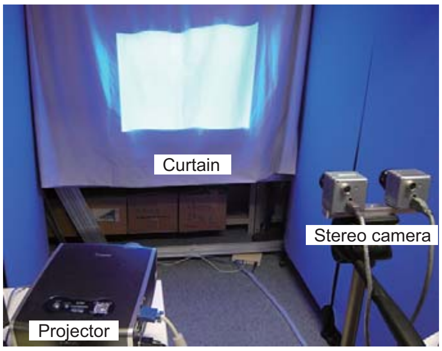


Fig. 4. Experimental environment.

system. The images used for the geometric correction are four video sequences: *Church*, *Harbor Scene*, *Buddhist Images* and *Flamingoes*. We compare the proposed method with the conventional method, i.e., the gray code pattern projection method.

Fig. 5 shows the 3D reconstruction result of the display surface obtained from the proposed method. This result indicates that we can measure the 3D shape of the display surface with high accuracy and dense by using 1D POC-based correspondence matching. Fig. 6 shows the projected

TABLE I
SPECIFICATION OF THE PC

| | |
|---|---|
| CPU | Intel Pentium 4 (3.0 GHz) |
| Memory | DDR2-SDRAM 2GB |
| GPU | Geforce 9800 GT |
| OS | Windows XP 32bit |

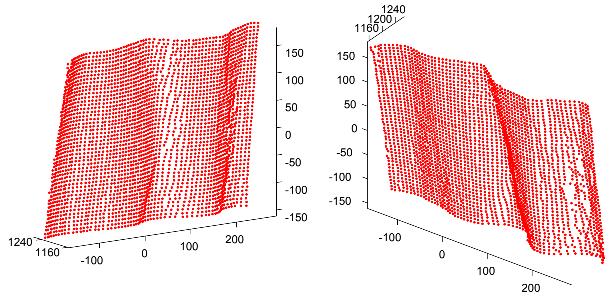

Fig. 5. 3D reconstruction result of the display surface.

image after geometric correction. Both methods can accurately correct geometric deformation of the projected image.

To evaluate the quantitative accuracy of the methods, the corners of the checkerboard pattern is used. Figs. 7 (c) and (d) show the projected image of the checkerboard pattern after geometric correction using the conventional and proposed methods, respectively, where the image transformation parameters used in geometric correction are the same as Fig. 6. And then, the coordinates of each corner are obtained by using MATLAB Camera Calibration Toolbox. The accuracy of geometric correction is evaluated as the differences between the true corner position and the measured corner position. Table II shows the RMS errors of corner position for conventional and proposed methods. This result indicates that the accuracy of the conventional and proposed methods is comparable.

The experimental results for other sequences are shown in Fig. 8. In the cases of changing the projected contents, the proposed method can correct the geometric deformation of the projected images. Depending on the contents, there are some areas having poor texture, which are hard to obtain accurate correspondence. This problem can be solves by applying the proposed method to the video sequences iteratively.

As is observed from the above experimental results, the proposed method can correct geometric deformation of the projected images without using structured light patterns. The proposed method needs only the prior stereo camera calibration. The focus and zoom settings of the projector can be changed, since the projector calibration is performed during projecting the contents. Also, when changing the shape of the display surface, the proposed method can correct deformation of the projected images using the projected contents. The use of the proposed method makes it possible to enhance convenience of projectors.

## V. Conclusion

This paper has proposed a geometric correction method using stereo camera for projected images. The use of the pro-

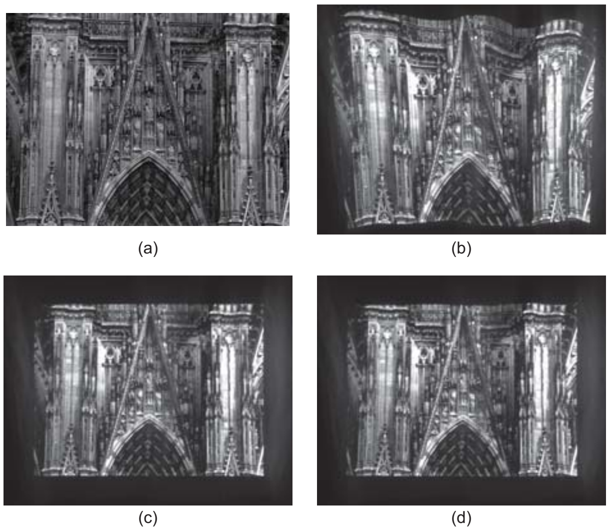


Fig. 6. Geometric correction result: (a) original image, (b) projected image without geometric correction, (c) projected image after geometric correction using structured light patterns, and (d) projected image after geometric correction using proposed method.

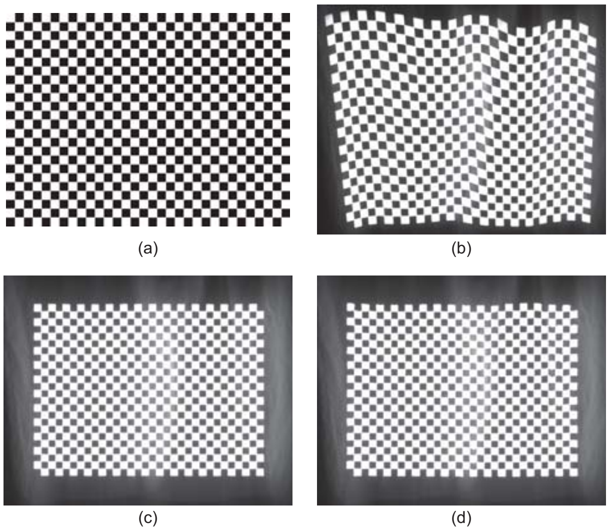


Fig. 7. Geometric correction result (projection of the checker pattern): (a) original image, (b) projected image without geometric correction, (c) projected image after geometric correction using structured light patterns, and (d) projected image after geometric correction using proposed method.

posed method makes it possible to correct geometric distortion of projected images during projecting the video contents. In future, we will develop the projector-camera system using the proposed method, which can correct the geometric distortion of the projected image in real-time.

TABLE II

ERROR OF THE CORNER POSITION AFTER GEOMETRIC CORRECTION.

| | Conventional method | proposed method |
|---|---|---|
| RMS Error | 1.48 pixels | 1.86 pixels |

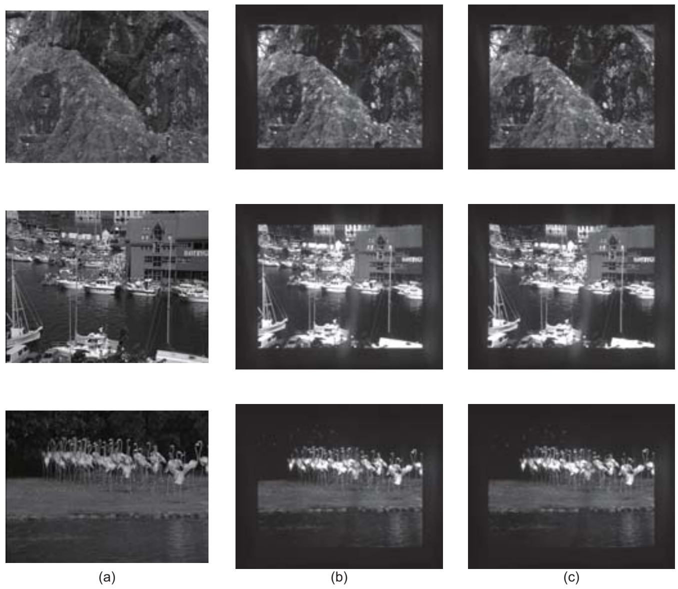


Fig. 8. Geometric correction result (top: *Buddhist Images*, middle: *Harbor Scene*, and bottom: *Flamingoes*: (a) original image, (b) projected image after geometric correction using structured light patterns, and (c) projected image after geometric correction using proposed method.

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