

## A Non-Rigid Registration Method for Medical Volume Data Using 3D Phase-Only Correlation

Yuichiro Tajima, Koichi Ito and Takafumi Aoki

Graduate School of Information Sciences, Tohoku University, Sendai-shi 980–8579, Japan.  
tajima@aoki.ecei.tohoku.ac.jp

### Abstract

*Volume registration is an essential technology for comparing medical volume data acquired in different days and for combining different types of volume data from various imaging devices. For accurate comparison, it is necessary to correct non-rigid deformation between volume data, since the complex deformation between medical volume data is observed even if they are taken from the same regions of the subject. This paper proposes a novel non-rigid registration method using 3D Phase-Only Correlation (POC). The proposed method achieves accurate and fast volume registration by employing POC-based block matching (with small volume blocks), which can find the voxel correspondence with sub-voxel accuracy between two volume data. The proposed method exhibits higher accuracy and shorter computation time compared with the conventional method, and is effective even for multimodality cases such as CT-MRI registration.*

### 1 Introduction

Registration between medical volume data such as Computed Tomography (CT) and Magnetic Resonance Imaging (MRI) is one of the important techniques in the field of medical image processing. The complex deformation between medical volume data is observed due to soft organs, imaging devices, temporal change of organs, etc. even if they are taken from the same regions of the subject. To compare volume data acquired in different days or combine different types of volume data from various imaging devices, the accurate and fast non-rigid volume registration method is required.

The most of conventional methods are based on maximization or minimization of the similarity measures such as Normalized Mutual Information (NMI) [4], Residual Complexity (RC) [3] and the Jensen-Tsallis

(JT) similarity [1]. These methods need to estimate parameters of the non-rigid deformation model by solving a large-scale nonlinear optimization problem. It is a very time-consuming task and is not suitable for the practical situation.

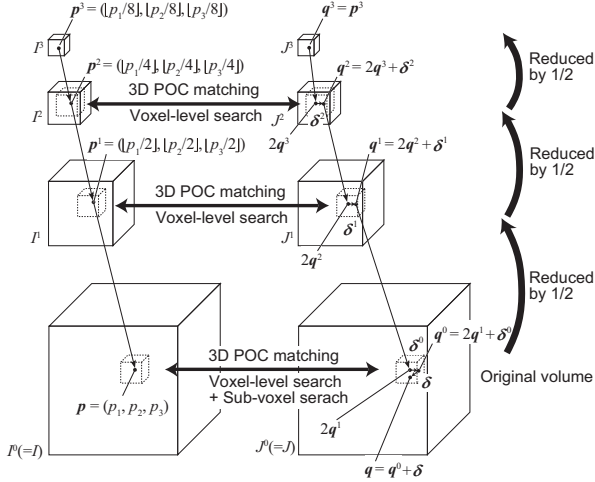
Addressing this problem, this paper proposes a novel volume registration method using 3D Phase-Only Correlation (POC) [5]. 3D POC is a volume matching technique using the phase components in 3D Discrete Fourier Transforms (DFTs) of given volumes. The proposed method consists of (i) correspondence matching using a coarse-to-fine strategy and 3D POC-based local voxel matching and (ii) deformation correction using B-spline model. Through a set of experiments using actually scanned CT and MRI data, we demonstrate the effectiveness of the proposed method compared with the conventional method.

### 2 Non-Rigid Registration Using 3D Phase-Only Correlation

The proposed method consists of the following 2 steps: (i) correspondence search using 3D POC and (ii) deformation correction using B-spline model. In the step (i), we obtain the correspondence with sub-voxel accuracy between the two volume data using the 3D POC-based correspondence matching. In the step (ii), we estimate the parameters of B-spline model using the correspondence obtained from the step (i), and then align the two volume data using the B-spline model with the estimated parameters. In the following, we describe the three key techniques of the proposed method such as (i) 3D POC, (ii) correspondence search using 3D POC and (iii) non-rigid deformation using B-spline.

#### 2.1 3D POC

We introduce the fundamentals of a 3D POC function, which is the extended version of 2D POC function [5]. Consider two volume data  $f(n_1, n_2, n_3)$  and



**Figure 1. Correspondence search using coarse-to-fine strategy for  $l_{max} = 3$ .**

$g(n_1, n_2, n_3)$ , and their 3D DFTs  $F(k_1, k_2, k_3)$  and  $G(k_1, k_2, k_3)$ . Then, the normalized cross-power spectrum  $R(k_1, k_2, k_3)$  is define as

$$R(k_1, k_2, k_3) = \frac{F(k_1, k_2, k_3)\overline{G(k_1, k_2, k_3)}}{|F(k_1, k_2, k_3)\overline{G(k_1, k_2, k_3)}|}, \quad (1)$$

where  $\overline{G(k_1, k_2, k_3)}$  denotes the complex conjugate of  $G(k_1, k_2, k_3)$ . The 3D POC function  $r(n_1, n_2, n_3)$  is the 3D Inverse DFT of  $R(k_1, k_2, k_3)$ . When two volume data are similar, their POC function gives a distinct sharp peak like the Kronecker delta function. The height of the peak can be used as a good similarity measure for volume matching, and the location of the peak shows the transnational displacement between the two volume data. We can estimate the true peak position with sub-voxel accuracy by fitting the analytical peak model of the 3D POC function to the calculated data array around the correlation peak as well as the 2D POC function [5].

## 2.2 Correspondence Search Using 3D POC

In order to correct non-rigid deformation between volume data, we have to find the accurate correspondence between volume data. We employ the correspondence search algorithm combining a coarse-to-fine strategy using image pyramids and local voxel matching using 3D POC. This algorithm finds the point  $\mathbf{q} = (q_1, q_2, q_3)$  on the volume data  $J$  corresponding to the reference point  $\mathbf{p} = (p_1, p_2, p_3)$  on the volume data  $I$  with sub-voxel accuracy. The followings are detailed procedure of this algorithm (Fig. 1).

**Step 1:** For  $l = 1, 2, \dots, l_{max}$ , create  $l$ -th layer volume

data  $I^l$  and  $J^l$ , i.e., coarser versions of  $I_0 (= I)$  and  $J_0 (= J)$  by recursively reducing  $I_0$  and  $J_0$  by  $1/2$ .

**Step 2:** For every layer, calculate the coordinate  $\mathbf{p}_l = (p_1^l, p_2^l, p_3^l)$  corresponding to the original reference point  $\mathbf{p}_0 (= \mathbf{p})$  recursively as follows:

$$\mathbf{p}^l = (\lfloor 2^{-l} p_1 \rfloor, \lfloor 2^{-l} p_2 \rfloor, \lfloor 2^{-l} p_3 \rfloor), \quad (2)$$

where  $\lfloor z \rfloor$  denotes the operation to round the element of  $z$  to the nearest integer towards minus infinity. We assume that  $\mathbf{q}^{l_{max}} = \mathbf{p}^{l_{max}}$  in the coarsest layer. Let  $l = l_{max} - 1$ .

**Step 3:** From  $l$ -th layer volumes  $I^l$  and  $J^l$ , extract two small 3D blocks  $f^l$  and  $g^l$  with their centers on  $\mathbf{p}^l$  and  $2\mathbf{q}^{l+1}$ , respectively. The size of 3D blocks is  $W \times W \times W$  voxels. In this paper, we employ  $W = 32$ .

**Step 4:** Estimate the displacement between  $f^l$  and  $g^l$  with voxel accuracy using 3D POC. Let the estimated displacement vector be  $\delta^l$ . The  $l$ -th layer correspondence  $\mathbf{q}^l$  is determined as follows:

$$\mathbf{q}^l = 2\mathbf{q}^{l+1} + \delta^l. \quad (3)$$

**Step 5:** Decrement the counter by 1 as  $l = l - 1$  and repeat from Step 3 to Step 5 while  $l \geq 0$ .

**Step 6:** From the original volumes  $I^0$  and  $J^0$ , extract two small 3D blocks with their centers on  $\mathbf{p}^0$  and  $\mathbf{q}^0$ , respectively. Estimate the displacement between the two blocks with sub-voxel accuracy using 3D POC. Let the estimated displacement vector with sub-voxel accuracy be denoted by  $\delta$ . Update the corresponding points as follows:

$$\mathbf{q} = \mathbf{q}^0 + \delta. \quad (4)$$

Also, the peak value of the 3D POC function is obtained as a measure of reliability in local block matching. In the proposed method, we set many reference points on  $I$  and find their corresponding points on  $J$ .

## 2.3 Non-Rigid Deformation Using B-spline

We employ B-spline to correct the non-rigid deformation between the two volume data. B-spline is represented by linear combination of the Gaussian-like functions called the B-spline basis functions. Let the center of the basis functions (knot) be  $\mathbf{t}_{m_1, m_2, m_3} = (t_{m_1}, t_{m_2}, t_{m_3})$ , where  $m_1 = 0, \dots, K_1 - 1$ ,  $m_2 = 0, \dots, K_2 - 1$ ,  $m_3 = 0, \dots, K_3 - 1$ , and  $K_1, K_2$  and  $K_3$  indicate the number of knots for each axis direction. We put  $\mathbf{t}_{m_1, m_2, m_3}$  in a lattice pattern with a spacing of  $h_1, h_2$  and  $h_3$  for each axis direction. Consider the real-number coordinate  $\mathbf{x} = [x_1 \ x_2 \ x_3]^T$ , the 3rd-order B-

spline basis function  $B_{m_1, m_2, m_3}(\mathbf{x})$  is defined by

$$B_{m_1, m_2, m_3}(\mathbf{x}) = b\left(\frac{x_1 - t_{m_1}}{h_1}\right) b\left(\frac{x_2 - t_{m_2}}{h_2}\right) b\left(\frac{x_3 - t_{m_3}}{h_3}\right), \quad (5)$$

where

$$b(x) = \begin{cases} (3|x|^3 - 6|x|^2 + 4)/6 & (|x| \leq 1) \\ -(|x| - 2)^3/6 & (1 \leq |x| \leq 2) \\ 0 & (2 \leq |x|) \end{cases}.$$

For convenience, we replace the index of knots  $(m_1, m_2, m_3)$  by  $j = m_1 + m_2 K_1 + m_3 K_1 K_2$ . The B-spline function  $T(\mathbf{x})$  is given by

$$T(\mathbf{x}) = \sum_{j=0}^{K_1 K_2 K_3 - 1} B_j(\mathbf{x}) \mathbf{a}_j, \quad (6)$$

where  $\mathbf{a}_j = [a_{j1} \ a_{j2} \ a_{j3}]^T$  indicates the weight (control point) for each knots.

To determine the B-spline function  $T(\mathbf{x})$ , the weight  $\mathbf{a}_j$  is estimated from the corresponding point pairs  $\mathbf{p}_i$  and  $\mathbf{q}_i$ , where  $i = 0, \dots, N - 1$  and  $N$  is the number of corresponding point pairs. The optimal weight  $\hat{\mathbf{a}}_j$  is obtained by minimizing the distance between the reference point after deformation  $\mathbf{p}_i + T(\mathbf{p}_i)$  and the corresponding point  $\mathbf{q}_i$  as follows:

$$\hat{\mathbf{a}}_j = \arg \min_{\mathbf{a}_j} \sum_i \|\mathbf{q}_i - (\mathbf{p}_i + T(\mathbf{p}_i))\|^2 \quad (7)$$

Using Tikhonov regularization, the least squares solution of the above equation is given by

$$\hat{\mathbf{a}} = (\mathbf{B}^T \mathbf{B} + \mu \mathbf{I}) \mathbf{B}^{-1} (\mathbf{Q} - \mathbf{P}), \quad (8)$$

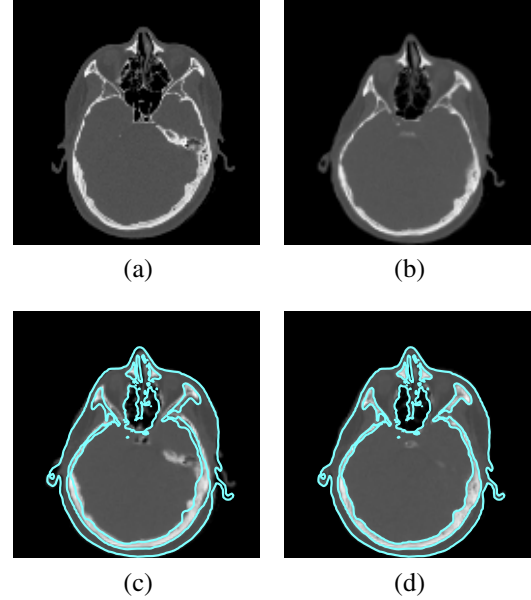
where  $\hat{\mathbf{a}} = [\hat{\mathbf{a}}_0 \ \dots \ \hat{\mathbf{a}}_{K_1 K_2 K_3 - 1}]^T$ ,  $\mathbf{P} = [\mathbf{p}_0 \ \dots \ \mathbf{p}_{N-1}]^T$ ,  $\mathbf{Q} = [\mathbf{q}_0 \ \dots \ \mathbf{q}_{N-1}]^T$ ,  $\mathbf{B}$  is the  $N \times K_1 K_2 K_3$  matrix with the element  $B_{ij} = B_j(\mathbf{p}_i)$ ,  $\mu$  is a coefficient. In the experiment, we employ  $\mu = 1$ . The coordinate  $\mathbf{n}'_I$  of the reference volume data  $I(\mathbf{n}_I)$  after registration is calculated by

$$\mathbf{n}'_I = \mathbf{n}_I + T(\mathbf{n}_I). \quad (9)$$

In this paper, we employ liner interpolation to generate the volume data after non-rigid registration.

### 3 Experiment

We evaluate the registration accuracy of the conventional and proposed methods using CT and MRI data. We use the non-rigid registration method proposed by Rueckert et al. [4] as the conventional method. This



**Figure 2. Registration results: (a) original CT data, (b) deformed data, (c) registration result using the conventional method and (d) registration result using the proposed method.**

method is based on maximization of NMI (Normalized Mutual Information) between volume data. The number of the knots for both methods is  $K_1 = K_2 = K_3 = 8$ .

We perform the quantitative evaluation using the original and deformed CT data. Let the original CT data be  $A(\mathbf{x})$  with  $128 \times 128 \times 128$  voxels and  $1.872 \times 1.872 \times 1.872$  mm resolution as shown in Fig. 2 (a). The deformed CT data  $B(\mathbf{x})$  as shown in Fig. 2 (b) is generated by

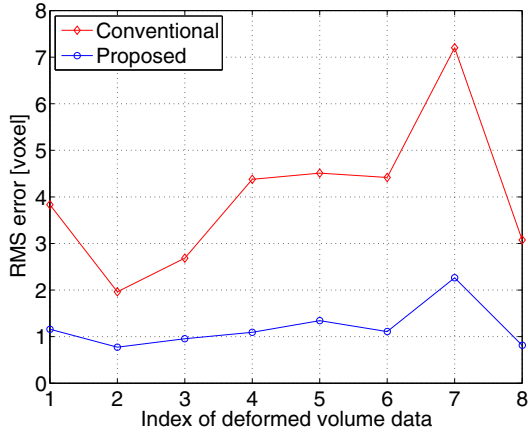
$$B(\mathbf{x}) = A(\mathbf{x} + \mathbf{T}_s(\mathbf{x})), \quad (10)$$

where

$$\mathbf{T}_s(\mathbf{x}) = \sum_{k=1}^{\nu} \alpha_k \exp(-\|\mathbf{x} - \beta_k\|^2 / 2\sigma_k^2), \quad (11)$$

where  $\nu = 16$ ,  $0 \leq \sigma_k \leq 64$ ,  $\alpha_k \in [-8, 8]^3$ ,  $\beta_k \in [1, 128]^3$ . By randomly changing  $\sigma_k$ ,  $\alpha_k$  and  $\beta_k$  within the above range, we obtain 8 deformed CT data  $B(\mathbf{x})$ . After non-rigid registration using the conventional and proposed methods, we evaluate the registration accuracy by Root Mean Square (RMS) error between the true deformation function  $\mathbf{T}_s$  and the estimated deformation function  $\mathbf{T}_e$  as follows:

$$\text{RMS} = \sqrt{\frac{1}{|C|} \sum_{\mathbf{n} \in C} \|\mathbf{T}_s(\mathbf{n}) - \mathbf{T}_e(\mathbf{n})\|^2}, \quad (12)$$



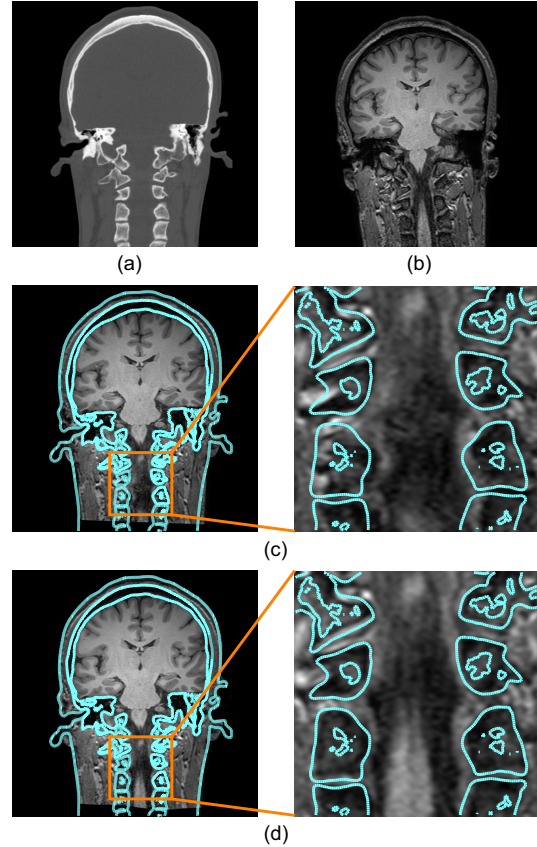
**Figure 3. RMS errors for the conventional and proposed methods.**

where  $C$  is a set of coordinates and  $|C|$  is the number of voxels within  $C$ . In this experiment, we set  $C$  to the region excluding the air from  $A$ . Figs. 2 (c) and (d) show the contour of deformed data  $B$  after registration and the slice image of the original data  $A$ . Fig. 3 shows RMS errors for each method. The conventional method cannot correctly align the CT data, while the proposed method can align deformed CT data with about 1-voxel error. The computation time is measured on Intel Xeon (3.00 MHz) with MATLAB 7.4 (64 bit). The average computation time of each method is 21,178 sec. for the conventional method and 163 sec. for the proposed method, respectively. From the above experiments, the proposed method exhibits higher accuracy and shorter computation time than the conventional method.

We apply the proposed method to multi-modal registration. We use CT and MRI volume data taken from the same subject as shown in Figs. 4 (a) and (b), where each data is  $512 \times 512 \times 512$  voxels and  $0.468 \times 0.468 \times 0.468$  mm resolution. Fig. 4 (c) shows the contour of CT data and the slice image of MRI data after rigid registration using 3D POC [2]. Fig. 4 (d) shows the result using the proposed method. Misalignment around the neck is observed in the case of rigid registration, while the proposed method can correctly align the CT and MRI data. As a result, the proposed method is also effective for multi-modal registration.

## 4 Conclusion

This paper has proposed a non-rigid volume registration method using 3D POC. Through a set of experiments, we demonstrate that the proposed method exhibits higher accuracy and shorter computation time than the conventional method. In future work, we will



**Figure 4. Registration result of CT and MRI data: (a) CT data, (b) MRI data, (c) result after rigid registration and (d) result after non-rigid registration.**

apply the proposed method to human identification using medical volume data.

## References

- [1] M. Khader and A. Hamza. Nonrigid image registration using an entropic similarity. *IEEE Trans. on Information Technology in Biomedicine*, 15(5):681–690, Sep 2011.
- [2] K. Miyazawa, Y. Tajima, K. Ito, T. Aoki, A. Katsumata, and K. Kobayashi. A novel approach for volume registration using 3D phase-only correlation. *Radiological Society of North America*, page 1070, Nov. 2009.
- [3] A. Myronenko and X. Song. Intensity-based image registration by minimizing residual complexity. *IEEE Trans on Medical Imaging*, 29(11):1882–1891, Nov 2010.
- [4] D. Rueckert, L. I. Sonoda, C. Hayes, D. L. G. Hill, M. O. Leach, and D. J. Hawkes. Nonrigid registration using free-form deformation: Application to breast MR images. *IEEE Trans. Medical Imaging*, 18(8):712–721, Aug 1999.
- [5] K. Takita, T. Aoki, Y. Sasaki, T. Higuchi, and K. Kobayashi. High-accuracy subpixel image registration based on phase-only correlation. *IEICE Trans. Fundamentals*, E86-A(8):1925–1934, Aug 2003.